VOLUME 21 JULY, 1933 NUMBER 7

PROCEEDINGS
of
The Institute of Radio Engineers

Form for Change of Mailing Address or Business Title on Page XVII

Institute of Radio Engineers Forthcoming Meetings

NEW YORK MEETING
October 4, 1933

PITTSBURGH SECTION
September 19, 1933

SEATTLE SECTION
September 29, 1933

E. R. Shute, Director, 1933

Emmett Ray Shute was born in Coal City, Indiana, on May 26, 1889. He received a B.S. degree in electrical engineering from Purdue University in 1912.

For a number of years prior to his graduation from college, he worked as a telegraph operator on various railroads in the middle west. In 1912 he joined the staff of the Western Union Telegraph Company as engineering assistant in the office of the vice president. He became an assistant traffic engineer in 1917 and general supervisor of traffic and costs in 1919. Since 1921 he has been operating engineer.

He is a member of the American Institute of Electrical Engineers, American Railway Association, and Radio Club of America. He joined the Institute of Radio Engineers as a Member in 1925.

June Meeting of the Board of Directors

The regular June meeting of the Board of Directors was held on the 7th at the Institute office and those present were L. M. Hull, president; Melville Eastham, treasurer; W. G. Cady, junior past president; O. H. Caldwell, Alfred N. Goldsmith, F. A. Kolster, E. L. Nelson, E. R. Shute, A. F. Van Dyck, William Wilson, and H. P. Westman, secretary.

A. L. Ainsworth, F. E. Gray, A. B. Oxley, L. P. Tuckerman, Marcel Wallace, and L. P. Williams were transferred from the Associate to the Member grade. In addition, thirty-one Associates, two Juniors, and sixteen Students were elected to membership.

In view of the early date on which the first Wednesday of September falls and the fact that no New York meeting is scheduled for September, the September meeting of the Board of Directors will be held on Wednesday, thirteenth.

Mr. Nelson, chairman of the Broadcast Committee, reported on some joint meetings which have been held by the engineering sections of the National Association of Broadcasters and the Radio Manufacturers Association, and the Institute Broadcast Committee. Considerable interest is indicated in an analysis of the broadcast situation from a systems viewpoint, and it is anticipated that from the coöperation of these three groups, important data can be obtained for the compilation of reports of interest to everyone in the broadcast field. The Board approved of the Institute's participation in this project and additional meetings will be held in the future.

In view of the difficulty of obtaining further contributions of sufficient magnitude to permit the continuance of survey work and the fact that almost as many members have obtained employment through Institute activities as have obtained assistance through survey work, it was agreed that surveys be discontinued, and the full efforts of the Emergency Employment Service be devoted to the obtaining of employment for those registered. This will permit a substantial reduction in operating cost. Those who have contributed in the past have done so with the knowledge that all money forwarded has been distributed to unemployed members in the purchasing of surveys. Contributions from those who are interested in assisting in the maintenance of this employment service will be appreciated, but no further appeals to the membership are planned.

The Institute was invited to be represented on a newly formed sub-

committee on vacuum tubes to operate under the Sectional Committee on Electrical Definitions. B. E. Shackelford, retiring chairman of the Institute's Standards Committee on Vacuum Tubes, was designated the Institute's representative.

Radio Transmissions of Standard Frequencies

The Bureau of Standards transmits standard frequencies from its station WWV, Beltsville, Md., every Tuesday. The transmissions are on 5000 kilocycles per second. On April 1, the schedule was changed. The transmissions are given continuously from 12 noon to 2 P.M., and from 10:00 P.M. to midnight, Eastern Standard Time. (From October to March, the schedule was from 10 A.M. to 12 noon, and from 8 to 10 P.M.) The service may be used by transmitting stations in adjusting their transmitters to exact frequency, and by the public in calibrating frequency standards, and transmitting and receiving apparatus. The transmissions can be heard and utilized by stations equipped for continuous-wave reception through the United States, although not with certainty in some places. The accuracy of the frequency is at all times better than one cycle per second (one in five million).

From the 5000 kilocycles any frequency may be checked by the method of harmonics. Information on how to receive and utilize the signals is given in a pamphlet obtainable on request addressed to the Bureau of Standards, Washington, D. C.

The transmissions consist mainly of continuous, unkeyed carrier frequency, giving a continuous whistle in the phones when received with an oscillating receiving set. For the first five minutes the general call (CQ de WWV) and announcement of the frequency are transmitted. The frequency and the call letters of the station (WWV) are given every ten minutes thereafter.

Supplementary experimental transmissions are made at other times. Some of these are made at higher frequencies and some with modulated waves, probably modulated at 10 kilocycles. Information regarding proposed supplementary transmissions is given by radio during the regular transmissions.

The Bureau desires to receive reports on the transmissions, especially because radio transmission phenomena change with the season of the year. The data desired are approximate field intensity, fading characteristics, and the suitability of the transmissions for frequency measurements. It is suggested that in reporting on intensities, the following designations be used where field intensity measurement apparatus is not used: (1) hardly perceptible, unreadable; (2) weak, readable now and then; (3) fairly good, readable with difficulty; (4)

good, readable; (5) very good, perfectly readable. A statement as to whether fading is present or not is desired, and if so, its characteristics, such as time between peaks of signal intensity. Statements as to type of receiving set and type of antenna used are also desired. The Bureau would also appreciate reports on the use of the transmissions for purposes of frequency measurement or control.

All reports and letters regarding the transmissions should be addressed to the Bureau of Standards, Washington, D. C.

Committee Work

Awards Committee

A meeting of the Awards Committee was held on April 25 at the office of the Institute, and was attended by William Wilson, chairman, W. G. Cady, J. V. L. Hogan, and H. P. Westman, secretary. The recommendations of the Committee were approved by the Board of Directors at its May meeting as indicated in the June Proceedings.

Constitution and Laws Committee

The Constitution and Laws Committee met on May 2 at the Institute office and proceeded with its revision of the Institute's constitution. Those present were J. V. L. Hogan, chairman, Austin Bailey, Arthur Batcheller, Melville Eastham, R. A. Heising, and H. P. Westman, secretary.

Convention Papers Committee

A meeting of the Convention Papers Committee was called for May 9, and was attended by only two members, E. L. Nelson and H. P. Westman, secretary. The technical papers program for the convention was put into tentative final form and copies forwarded to the other members of the committee for any comments or criticisms.

Nominations Committee

The Nominations Committee met on April 26 at the Institute office, and those in attendance were W. G. Cady, chairman, H. F. Dart, R. A. Heising, Donald McNichol, and H. P. Westman, secretary. As indicated in the June Proceedings, its recommendations were presented to the Board at the May meeting of that body.

Sections Committee

A meeting of the Sections Committee was held on April 25 in the Institute office, and those present were B. E. Shackelford, chairman, Austin Bailey, I. S. Coggeshall, H. C. Gawler, C. W. Horn, R. H.

Langley, W. D. Loughlin, J. H. Miller, and H. P. Westman, secretary. The meeting was devoted chiefly to a discussion of material to be considered at the annual meeting of the Sections Committee to be held at the Hotel Sherman, Chicago, Illinois at 4:00 P.M. on Tuesday, June 27.

Institute Meetings

Atlanta Section

A meeting of the Atlanta Section was held on February 9 at the Atlanta Athletic Club with Chairman H. L. Wills presiding.

A paper on "Time Delay in Communication Networks" was presented by J. R. Jessie of the American Telephone and Telegraph Company. In it, the author discussed in detail the time delays experienced in the transmission of electrical energy over telephone circuits. The paper was discussed by Messrs. Bangs, Gardberg, Gerks and Wills.

At a meeting of the Executive Committee which is comprised of H. L. Wills, H. L. Reid, P. C. Bangs, and H. F. Dobbs, I. H. Gerks was appointed chairman of the Meetings and Papers Committee; R. R. Brewin, of the Membership Committee; and H. L. Reid, as chairman of the Publicity Committee.

The attendance totaled thirteen and eight were present at the informal dinner which preceded the meeting.

The March meeting of the Atlanta Section was held at the Atlanta Athletic Club on the 9th with J. Gardberg as acting chairman.

At this meeting, N. V. Fowler of the American Telephone and Telegraph Company presented a paper on "Transmission Lines." The effect of the length of a wire line and atmospheric conditions upon the velocity of propagation of electrical waves along it were discussed. The electrical characteristics of lines were covered and the effect of the proximity of the line to other structures upon the losses in the line described. The effectiveness of both single-wire and double-wire transmission lines at radio frequencies was treated.

The paper was discussed by Messrs. Bangs, Daugherty, Eubanks, Gardberg, and Gerks of the fourteen members and guests in attendance.

Boston Section

A meeting of the Boston Section was held on March 10 at Harvard University, E. L. Chaffee, chairman, presiding.

The subject of the meeting was a discussion of "Applications of Relaxation Oscillations in Resistance-Capacitance Circuits to Radio

Engineering Problems" which was led by Messrs. J. K. Clapp, F. D. Hunt, and H. R. Mimno. A brief survey was given of the literature on relaxation oscillations and their application to frequency demultiplication, sweep circuits, stroboscopes, and pulsing circuits for Kennelly-Heaviside layer height determinations. A general discussion ensued. The attendance totaled 125.

The April meeting of the Boston Section was held on the 21st at Harvard University. E. L. Chaffee presided, and the attendance was 100.

J. R. Nelson of the Raytheon Production Corporation presented a paper on "Characteristics of the New Vacuum Tubes." In it, the author presented a survey of the characteristics and uses of some of the new designs of vacuum tubes recently placed upon the market together with a critical examination of their particular uses and advantages. The discussion was participated in by Messrs. Browning, Chaffee, Mimno and others.

Buffalo-Niagara Section

The Buffalo-Niagara Section held a meeting on April 20 at the University of Buffalo. In the absence of the chairman it was presided over by G. C. Crom, Jr.

A paper on "Broadcast Technique from the Microphone to the Antenna" was presented by E. L. Nelson of the Bell Telephone Laboratories.

The speaker pointed out some of the limitations and weaknesses of carbon-grain and condenser microphones, and then described the dynamic microphone. He stated that its use had disclosed defects in acoustic design in some studies which were apparently not evident with the use of older types of microphones.

Speech input apparatus and circuits operating from alternating-current power supplies were covered. He then described some low power transmitting equipment mounted in a convenient sized cabinet and operated from 110 volts or 220 volts alternating-current power supply. Grid bias modulation was employed so that the power could be increased by adding cabinets of similar proportions containing appropriate amplifiing equipment.

The quartz crystal control unit designed to fit into the main transmitter cabinet was then described. Design details of the constant temperature oven were discussed, and it was pointed out that in addition to a careful design of thermostat, heater, and thermo-insulation, wide variations in the temperature of the room are compensated for by

placing a heat-conducting piece of metal in contact with the thermostat and passing it through the wall of the enclosure.

Monitoring equipment was discussed in detail. The speaker then described the 50-kilowatt transmitter of KSL which is installed on the Salt Lake bed at Salt Lake City, Utah. Some of the transmitting apparatus at WHAN, WHAS and several other stations were covered.

The problem of synchronizing transmissions of two broadcast stations by supplying a reference frequency of 4000 cycles to each station over wire lines was considered, and it was pointed out that it was possible to maintain station frequencies within thrity electrical degrees when operating on a transmission frequency of the order of a million cycles. At the conclusion of the paper, a general discussion followed which was participated in by Messrs. Crom, Kingsley, Nelson and others.

The May 18 meeting of the Buffalo-Niagara Section was held at the University of Buffalo with L. Grant Hector, chairman, presiding. The members and visitors in attendance numbered seventy-one.

"Recent Developments in Short-Wave Technique" was the subject of a paper by H. N. Kozanowski, research physicist in the power tube laboratory of the Westinghouse Electric and Manufacturing Company plant at East Pittsburgh, Pa.

In discussing the problem of generating short waves, the speaker outlined the inherent limitations of three-electrode tubes. The first point he covered was the minimum value of inductance and capacity of a tuned circuit which would give the shortest wavelength at which any particular tube would oscillate. The second item considered the intrinsic stability of such a circuit involving the ratio of circulating or stored energy to the energy absorbed from the circuit in a radio-frequency cycle which is sometimes referred to as decrement, kva to kw ratio, or flywheel effect. The third consideration was of the time of flight of electrons from the cathode to the anode of a tube under the influence of the accelerating voltage. This governs the phase angle between the plate current and plate voltage and thus, the efficiency. In connection with this last limitation, it is important to note that the allowable heat dissipation in the tube governs the output directly.

In discussing the first limitation, the desirability of minimum inductance and capacity in the tube structure was pointed out. Tube elements should be small and widely spaced. The UX-852 and the RCA-846 have these features, but on account of the distance between plate and filament, the time of flight of electrons is appreciable (10^{-9} seconds) making the minimum plate voltage rather high. Instead of

using a circuit of lumped inductance and capacitance, a Lecher system which behaves as a transmission line with uniformly distributed constants may be employed. When its physical length corresponds to a quarter wavelength or some multiple, it behaves as a parallel tuned circuit. Shorter waves with higher output and efficiency are thus made possible.

By designing the tube and Lecher system to be equivalent to a continuous concentric pipe transmission line, the tube is electrically indistinguishable from the line and outputs of 20 kilowatts have been obtained at six meters, compared with maximum values of one kilowatt for other types of tubes.

Electron oscillators developed by Barkhausen and Kurz were then discussed, and it was pointed out that they were in general limited to very small powers. Some experiments with these types of oscillators were outlined.

A tube developed consisting of a straight axial filament and a two-sector cylindrical anode located in a magnetic field was then considered. When the magnetic field is parallel to the filament, oscillations of the negative resistance type developed by A. W. Hull are obtained. However, by inclining the magnetic field about five degrees, a new type of oscillations is developed which have been called magnetostatic oscillations. A forty-centimeter oscillator has developed powers as great as ten watts with an efficiency of eight per cent. By using a small anode diameter and stronger magnetic field the wavelength can be decreased, and approximately two watts have been obtained at eighteen centimeters, and one watt at nine centimeters.

Messrs. Burbank, Crom, Hayes, Huntsinger, Waud, Wesselman and others participated in the discussion.

Chicago Section

A meeting was held on January 30 in the Western Society of Engineers auditorium which was jointly attended by members of the Chicago Section of the Institute and the Telephone, Telegraph, and Radio Section of the Western Society of Engineers. R. D. Jessop, chairman of the latter group, presided. A paper on "The Cathode Ray Tube as an Engineering Instrument" was presented by R. M. Arnold, chairman of the Institute's Chicago Section.

The speaker outlined the constructional details and operating characteristics of cathode ray tubes and treated their use as electrical measuring devices. Complete cathode ray tube equipment was available and used to demonstrate the functions of the tube especially as an oscillograph. The attendance totaled 275.

The March meeting of the Chicago Section was held on the 29th. R. M. Arnold, chairman, presided. The attendance was 100.

"Recently Developed Radio Receiving Tubes" was the subject of a paper by F. H. Engel of RCA Radiotron Company. In it, the speaker discussed a substantial group of new tubes which have recently been developed and placed on the market. Detailed descriptions were given of the various tubes, their operating characteristics were outlined, and their usefulness in various circuit arrangements treated.

On April 21, the Chicago Section met. The presiding officer was R. M. Arnold, and the attendance was sixty-five.

W. J. Polydoroff of the Johnson Laboratories presented a paper on "Ferro-Inductors and Permeability Tuning in Broadcast Receivers." As the paper appeared in full in the May issue of the PROCEEDINGS, it is not summarized here.

CINCINNATI SECTION

The University of Cincinnati was the place of the March 14 meeting of the Cincinnati Section which was presided over by W. C. Osterbrock. The attendance totaled sixty.

"The Design, Characteristics, and Uses of the New Electromagnetic Microphones" was the subject of a paper by Stanford Goldman. He reviewed briefly design types of microphones, placing emphasis on the condenser type and its inherent weakness for sound picture use. A description of the ribbon type microphone followed and the elementary theory of its operation, its mechanical advantages and special uses covered. The dynamic type of microphone was then treated and an electrical analogy presented of its mechanical characteristics. Means for obtaining a linear characteristic between the acoustical input and electrical output were indicated by means of this analogy.

The paper was closed with a brief description of special types of microphones for directional pickup.

Messrs. Aughenbaugh, Knoblaugh, Osterbrock, Rockwell, and others participated in the discussion.

A meeting of the Cincinnati Section was held at the University of Cincinnati on April 18 and was presided over by W. C. Osterbrock, chairman. Sixty-five members and guests attended.

"The Intermittent Glow Discharge and its Application to the Production of Music" was the subject of a paper by Winston Kock. The speaker opened the paper with a discussion of the electrical production in music in general and the specific types of instruments which have

been developed. He followed that with a general discussion of glow discharges and the possibilities of their application to electrical musical instruments. The difficulties encountered in attempting to complete a neon tube organ where space requirements and pitch constancy were important was discussed. It was found possible to construct such a device by using a resonance circuit in conjunction with a neon tube oscillator and a tuned circuit in the amplifier to introduce harmonics and improve the quality of the tone. The paper was concluded with a discussion of the Hallformant theory and from tests made it was concluded that this theory does not hold true for this particular organ.

Cleveland Section

The Cleveland Section held a meeting at Case School of Applied Science on April 28. P. A. Marsal, chairman, presided and the meeting was attended by thirty-six members and guests.

"Radio—Its Development and Use by the Navy" was the subject of a paper by L. L. Becker, Lieutenant in the United States Navy. In it he traced briefly the developments of electrical means of communication and the great interest of the Navy in this subject. He listed as the principal motives of the United States Navy in developing radio communication the desirability of insuring communications between the fleets and Navy Department headquarters, protection of life and property at sea, and the desirability of making the United States self-sustaining in the communications field.

He pointed out that a major part of the radio activities in the United States from 1903 to 1919 were centered in the Navy, and that since 1919 radio has been developed for widespread naval, military, and civilian uses.

Lieutenant Becker then outlined the development of spark transmitters at Arlington Navy Station from 100 watts to 100 kilowatts in 1915 and the later replacement of the spark by a 3-kilowatt arc which proved more effective. Following these tests, arc transmitters up to 500 kilowatts were ordered for naval installations. The evolution of efficient tube transmitters of comparatively low power was given as the reason for the Navy's complete change to that type of equipment and the resulting obsolescence of about $15,000,000 worth of previous equipment".

The speaker then discussed the radio compass which proved a valuable wartime instrument and the parent of the radio airways beacon which has proved so valuable. A formal discussion followed the presentation of the paper.

Connecticut Valley Section

A meeting of the Connecticut Valley Section was held on March 23 at the Hotel Garde in Hartford, Conn., and was presided over by H. W. Holt, chairman.

J. J. Lamb, technical editor of *QST*, presented a paper on "Practical Use of Highly Selective Single Radio-Frequency Circuits." In it, he outlined the development of receivers in which were incorporated circuits possessing such extreme selectivity as to eliminate the audio-frequency image in continuous-wave telegraph reception as well as to enable the intelligible reception of voice modulated signals under conditions of extreme interference. Two receivers employing different types of high selectivity circuits were described and demonstrated. One had a piezo-electric quartz crystal filter in a suitably balanced circuit with provision for series and parallel connections, while a simpler receiver employed a regenerative amplifier which gave pronounced selectivity and sensitivity when operated on the verge of oscillation. A special circuit and suitable precautions were employed to keep the amplifier stable despite the high order of regeneration employed.

A number of the thirty-two members and guests in attendance participated in the discussion of the paper.

Committee chairman appointments were announced. C. J. Madsen will head the Membership Committee; Reuben Lee, the Meetings and Papers Committee; and M. E. Bond will be chairman of the Publicity Committee.

The April meeting of the Connecticut Valley Section was held on the 20th at the Hotel Charles in Springfield, Mass. H. W. Holt, chairman, presided, and the attendance was fifteen.

"The Technique of Broadcasting" was the subject of a paper by Mr. Holt who is chief engineer of WMAS of Springfield.

In it, he discussed the development and operation of a modern broadcast station from the engineer's viewpoint. The mechanics of broadcasting were covered ranging from the electrical details of the operating portions to the placing of artists, sound effects, announcer-operator signals, remote control pick-ups, and the like. The characteristics and use of modern live-end and dead-end studios were covered. Some interesting observations were made concerning the use and placement of a battery of two or more microphones and the effect of the varying phase relationships at different frequencies.

Detroit Section

On April 18 a joint meeting of the Detroit-Ann Arbor Section of the American Institute of Electrical Engineers and the Detroit Section of

the Institute was held at the Detroit Edison Auditorium. O. E. Hauser, chairman of the A.I.E.E. section, presided, and six hundred were present. "Modern Theories of the Composition of Matter" was the subject of an address by A. H. Compton, Professor of Physics at the University of Chicago.

The speaker outlined some recent experiments which tended to show the existence not only of the electron and proton but of additional particles of matter called neutrons and positrons.

As fine drops of water in clouds or fog are always formed around a speck of dust these phenomena could be formed in a glass chamber fitted with a piston which when drawn out would expand the moist air in the chamber. The water particles would condense not only upon specks of dust but upon ions or other particles in the chamber, and the paths which these particles pursued could be photographed by the effects on the moist air. By subjecting these particles to the effects of electric and magnetic fields the paths of the ions could be changed, and their electrical polarity, size, and mass could be computed.

Dr. Compton closed his paper with a discussion of the wave characteristics of the electron indicating their ability of being defracted. Similar effects could be obtained with protons by using a metal plate in place of the regular type of defraction grating. A general discussion followed the paper.

The May meeting of the Detroit Section was held at the Detroit News Conference Room on the 19th with G. W. Carter, chairman, presiding.

N. H. Williams, Professor of Physics at the University of Michigan, presented a paper on "Piezo-Electric Crystals." In it he gave a résumé of the fundamental points of piezo electric action and, neglecting other materials, concentrated on quartz because of its interest in the radio field. By means of models and slides, right-hand, left-hand, and twin crystals were illustrated, and the optical and electrical characteristics of each outlined.

After discussing the electrical, mechanical, and optical axes of crystals and methods of cutting them, the speaker pointed out a number of ambiguities that have arisen in the literature on the subject. He showed that the electric and X-axes are the same, and that the zero-degree cut, Curie-cut, X-cut, and perpendicular-cut are identical. He also pointed out the difficulties due to changes in constants from one crystal to another and showed that the X-cut for one mode of vibration becomes the Y-cut for another.

Professor Williams then showed that it was possible to make a

crystal vibrate at a harmonic frequency by mechanically exciting the plate with an air jet and reading the voltage generated in the crystal with an amplifier and a small contact which is moved across the face of the plate by a micrometer screw. A curve drawn with the measuring amplifier tuned to the fundamental frequency showed humps indicating the presence of harmonics; when the amplifier was tuned to various harmonics, voltage curves of the output were drawn. It was found that by using a push-pull circuit with two properly placed electrodes it was possible to make a crystal oscillate electrically at its second harmonic. Also, by proper placement and the correct number of electrodes it was possible to obtain oscillations with frequencies up to the ninth harmonic. The paper was closed with a brief discussion of the use of Rochelle salt crystals for loud speakers.

A number of the fifty-five members and guests participated in the general discussion.

Los Angeles Section

On March 21 a meeting of the Los Angeles Section was held at the Hotel Arcady with J. K. Hilliard, chairman, presiding.

R. G. Leitner presented a paper on "Radio Receivers for Home Entertainment." In it he discussed the radio engineering field from the angle of cost vs. quality and the demands on the engineer to meet present competition. The development of the midget type receiver from a comparatively ineffective device to its present extremely low cost status was outlined. The development of audio-frequency power tubes during the last few years was then discussed, and some pertinent remarks made regarding the necessary and desirable output power for home receivers. Class B audio amplification and tubes therefor together with some of the newer multiuse and multielement tubes were described and their merits mentioned. The desirability of developing more efficient loud speakers for better fidelity of reproduction in moderately priced receivers was expressed.

Advantages and disadvantages of automatic volume control and methods of obtaining it were discussed as were tuning meters, a quiet automatic volume control, and spot tuning.

The speaker then covered the subject of oscillators, outlining developments in this field and proceeding to the five-grid mixer tubes recently developed for superheterodyne receivers.

The second paper of the evening on "Aircraft Receivers" was presented by Paul O'Connor who pointed out three groups into which aircraft radio uses fall. These are short-wave communication, long-wave communication, and radio beacon service. Details concerning

each of these services were given briefly, and it was pointed out that in all aircraft radio work such characteristics as reliability, weight, and sensitivity were of major importance.

The speaker then covered the subject of power supply, mechanical design, and antenna considerations indicating the various trends and requirements of the different branches of the industry. Motor noise suppression was covered. Past, present, and future types of vacuum tubes for aircraft use were discussed, and it was pointed out that automatic volume control was not suitable for aircraft radio purposes in general. The high cost of such installations was given as the reason why few private owners have employed radio. A brief description of air transport equipment ended the paper, and the general discussion of both papers was participated in by many of the sixty-five members and guests in attendance.

Nineteen were present at the informal dinner which preceded the meeting.

The Los Angeles Junior College was the place where the April 18 meeting of the Los Angeles Section was held. J. K. Hilliard, chairman, presided and the attendance was 125.

Three papers were presented, the first of which on "Single-Signal Superheterodynes" was by Ralph Gordon who opened his paper with a brief definition of selectivity and a description of various forms of it. He then outlined an ideal superheterodyne receiver as concerns the necessary and desirable number of stages of radio-frequency and intermediate-frequency amplification. Some pertinent remarks covering the underlying theory and basic development of tuned circuits and superheterodyne receivers followed. Mr. Gordon then traced the development and operation of a single-signal superheterodyne from the original introduction of the quartz filter some years ago to its present state giving some results obtained with experimental models.

The necessity of supplying separate gain controls on the radio-frequency and intermediate-frequency amplifiers, the unsuitability of screen-grid tubes as second detectors, and the undesirability of high grid lead capacity were covered. The paper was concluded with a discussion of crystals best suited for this purpose and of switching arrangements to permit the crystal filter to be used in several ways.

The second paper by C. O. Perrine covered "High Efficiency Radio-Frequency Power Amplifiers." The speaker discussed several methods of measuring the power output of radio-frequency power amplifiers and estimated that the method employed in his work was good to within about four per cent. He then proceeded to discuss suitable amplifier

circuits and outlined the development of one giving high power output when the type of tube employed was considered. He recommended the use of large antenna inductance, the careful adjustment of excitation voltage, and high plate voltage. The tubes were biased as class C amplifiers to obtain high circuit efficiency.

The final paper by D. C. Wallace was on "Short-Wave Antennas and Transmission Lines." In it were discussed several antennas of different types which had been constructed and employed over a period of years. Tabulations were presented giving the various results of these antennas both for receiving and transmitting. An antenna for harmonic operation was discussed in detail. The speaker then discussed the advantages of directional antennas. A description was given of portable transmitters and temporary antenna installations in hotels.

All three papers were discussed by many of those in attendance.

New York Meeting

The New York meeting of the Institute was held on May 3 in the Engineering Societies Building, President Hull presiding.

Two papers were presented, the first being "A Radio Beacon Free from Night Effects" by Howard A. Chinn of the Columbia Broadcasting Sytem. In it, the author described a radio range beacon, suitable for the guidance of aircraft, along established airways, which is entirely free from atmospheric variations or "night effects." Advantage was taken of the phenomenon that waves of frequencies higher than 30 megacycles per second, or thereabouts, are not usually refracted back to the earth by the Kennelly-Heaviside layer. Multiple-path transmission, variation in signal intensity and in polarization are thus avoided. A four-course aural beacon operating on 34.6 megacycles per second was employed for the experimental work. Results and applications were discussed.

The second paper, "On the Solution of the Problem of Night Effects with the Radio Range Beacon System," was presented by Harry Diamond of the Bureau of Standards. A new antenna system was described for use at radio range beacon stations which eliminates the troublesome night effects hitherto experienced in the use of the range beacon system. Considerable data, comprising ground and flight measurements, were given on both aural and visual type range beacons using the present loop transmitting antennas, which show the severity of the night effects encountered. Because of the magnitude of these effects, particularly in mountainous country, the range beacon course often becomes of no value beyond about thirty miles from the beacon station. With the new antenna system developed, referred to as the

transmission-line antenna system, the beacon course is satisfactory through its entire distance range, the night effects becoming negligible. Experimental data were given comparing the performance of the transmission-line and loop antenna systems under nearly identical conditions. The paper included a theoretical analysis of the phenomena underlying the occurrence of night effects and of how to eliminate them.

Several of the 200 members and guests who attended the meeting participated in the discussion which followed.

Philadelphia Section

H. W. Byler, chairman, presided at the April 6 meeting of the Philadelphia Section held at the Engineers Club in Philadelphia.

At this meeting awards were announced of Student memberships in the Institute to the two undergraduate students attending colleges in the Philadelphia Section territory whose papers were considered best of those submitted to the Meetings and Papers Committee of the Section.

The first prize, a three-year membership and an Institute emblem, was presented to J. G. Haines, a student at Haverford College for his paper on "Development of Communications in the Field of Ultra-Short Waves." The second prize was a one-year membership and pin and was presented to S. C. Spielman, who is studying at the Drexel Institute, for his paper on "A System Utilizing Radio Signals for the Blind Landing of Aircraft."

Mr. Haines presented his paper as the technical portion of the meeting. In it, the developments of ultra-short-wave communication covering investigations of workers to the present time were outlined. A general discussion followed which was participated in by a number of the sixty-seven members and guests in attendance.

At the close of the meeting, announcement was made of the slate of candidates for office for the ensuing year.

Pittsburgh Section

R. T. Griffith, chairman, presided at the April 24 meeting of the Pittsburgh Section held at the Fort Pitt Hotel.

"Remote Control Devices Applied to Radio Receivers" was the subject of a paper presented by Mr. Griffith, and considerable discussion ensued at its close.

A slate of officers for election at the May meeting of the section was submitted by the Nominations Committee and accepted. The attendance totaled nineteen.

The annual meeting of the Pittsburgh Section was held on May 23 with R. T. Griffith, chairman, presiding. This was a dinner meeting at which no technical paper was presented. In the election of officers, Lee Sutherlin of the Westinghouse Electric and Manufacturing Company became chairman; C. K. Krause of the Duquesne Light Company, vice chairman; and A. P. Sunnergren, West Penn Power Company, secretary-treasurer. The attendance totaled twenty-two.

San Francisco Section

The San Francisco Section held a meeting on April 18 at the Bellevue Hotel with A. R. Rice, chairman, presiding.

"The New Fifty-Thousand Watt Broadcast Transmitter at KPO" was the subject of a paper by E. J. Frost, radio engineer of the General Electric Company. In it there was covered not only the design and construction of this transmitter but also the subject of its adjustment after installation. A rather general and lengthy discussion resulted which was participated in by Messrs. Heintz, Royden Terman, Whitton and several others of the seventy-five who attended the meeting. Twenty were present at the informal dinner which preceded it.

The May meeting of the San Francisco Section was held at the Bellevue Hotel with the Chairman A. R. Rice presiding.

A. H. Halloran, television consultant, presented a paper on "Tomorrow's Television Equipment." In it he considered to a large extent the future possibilities of television. At the conclusion of the talk the seventy-five members and visitors in attendance proceeded to the Television Laboratories, Inc., where a demonstration using both direct pick-up and film pick-up was given with synchronized sound. The demonstration was successful, and all present were impressed with the progress made in this important branch of the radio art. The discussion of the paper was participated in by Messrs. Cohen, Heintz, Pratt and several others.

Seattle Section

A meeting of the Seattle Section was held on March 31 at the University of Washington with R. C. Fisher, vice chairman, presiding.

"Vacuum Tube Voltmeters" was the subject of the paper by Abner R. Willson who first reviewed the literature on the subject and then discussed the characteristics of the various circuit arrangements and pointed out their limitations. The use of such instruments for measuring the performance of radio- and audio-frequency circuits was covered in detail and demonstrated by suitable equipment. Many other appli-

cations were than cited to indicate the versatality of the vacuum tube voltmeter for measuring minute potential differences as well as those in the order of hundreds of thousands of volts. The necessity for using gas-free tubes was stressed and the considerable error which the reversed grid current of a slightly gassy tube introduced was discussed.

The attendance at the meeting totaled seventy-five. The discussion which followed the presentation of the paper was participated in by Messrs. Fisher, Libby, Mossman, Morris and others.

The April meeting of the Seattle Section was held on the 29th at the University of Washington and was presided over by H. H. Bouson, chairman.

The paper of the evening, "Recent Developments in Electrical Musical Instruments," was presented by C. E. Williams who introduced the subject by outlining the experimental work being done both in Europe and America on various forms of electrical and musical instruments. He described a number of different kinds of such instruments and discussed the problems encountered in the tuning of these to the musical scales.

The main portion of the paper was devoted to a description and demonstration of various features of the "Claribass," a five-octave keyboard instrument resembling an organ and employing vacuum tube tone generators. This instrument was developed and constructed by the speaker. Experimental equipment was used to demonstrate the possibility of keying such an instrument by conductance through the hand and fingers of the player and means for obtaining the desired tonal pitch by selection of the proper contact with the keys. Wave forms of notes both in and out of tone with one another was shown on a large screen by means of an oscillograph.

A comprehensive outline was given on the subject of vibrato and its relation to musical and psychological effects as well as to timbre. Means for producing vibrato in its various forms was demonstrated. A musical program of several selections closed the meeting which was attended by 100 members and guests.

Toronto Section

The April meeting of the Toronto Section was held on the 21st at the University of Toronto with R. A. Hackbusch, chairman, presiding.

"Modern Broadcast Transmitters from Microphone to Antenna" was the subject of a paper by E. L. Nelson of the Bell Telephone Laboratories. In it, he explained details of dynamic microphones and speech input equipment which he pointed out is now entirely operated

from alternating-current supplies. He showed oscillograms of antenna current at one hundred per cent modulation and also characteristic curves of a typical one-kilowatt transmitter. Complete details of crystal control units were given and the features of a number of fifty-kilowatt transmitters explained. The vertical antenna system recently installed at WABC was discussed in detail.

Messrs. Mott, Murphy, Shane and others of the sixty-six members and guests in attendance participated in the discussion.

The annual meeting of the Toronto Section was held on May 17 at the University of Toronto and was called to order by R. A. Hackbusch, retiring chairman. In the election of officers which immediately followed, W. F. Choat of the Canadian Westinghouse Company was elected chairman; R. C. Poulter, editor of *Radio Trade Builder*, vice chairman; G. E. Pipe of the Rogers Majestic Corporation, secretary-treasurer, and A. B. Oxley of Philco Products of Canada was elected acting secretary. The retiring chairman reviewed the activities of the section during the past year and a final report was submitted by the secretary-treasurer.

The paper of the evening, "New Vacuum Tubes and Their Applications," was by W. H. Kelterborn and T. S. Farley of the Canadian Westinghouse Company. The first part of it was presented by Mr. Kelterborn who reviewed developments and construction of new tubes including such devices as the dome-shaped bulb and new types of cathodes. He outlined the characteristics of some of the more recent tubes and manufacturing processes were illustrated by means of slides.

Mr. Farley in his portion of the paper dealt with the application of the newer types of tubes from the set engineer's viewpoint. In it he gave the main characteristics of the tubes under actual working conditions and discussed circuit requirements for most efficient operation.

The extensive discussion of the paper entered into by Messrs. Fox, Hackbusch, Nesbit, Price and others indicated the great confusion which has been caused by the introduction of so many new tubes. It was felt that design problems would be simplified by the adoption of but a few useful designs which could be concentrated on to permit reasonably high efficiency to be obtained in their use.

Sixty-five members and guests attended the meeting.

Washington Section

A meeting of the Washington Section was held on April 13 at the Kennedy-Warren Apartments Hotel and was presided over by H. G. Dorsey, chairman.

F. A. Cowan, engineer for the American Telephone and Telegraph Company, presented a paper on "Telephone Circuits for Program Transmission." He described the arrangement of telephone circuits employed in linking together a great number of broadcast stations affiliated with the two major networks. The necessary precautions taken to prevent noise from affecting such circuits and the attenuation necessary to provide substantially flat characteristics was given in detail. Many practical problems encountered in the operation of program transmission circuits were outlined. The manner of changing the direction of program transmission according to the origin of the program and the apparatus employed for pre-selection of different connections of stations was illustrated and described.

Messrs. Burgess, Dorsey, Lyons and others of the forty-six members and guests in attendance participated in the discussion of the paper. Sixteen attended the dinner which preceded the meeting.

The May meeting of the Washington Section was held on the 11th at the Kennedy-Warren Apartments Hotel with Dr. Dorsey presiding.

"The Radio Art in the U.S.S.R." was the subject of a paper by Louis Cohen who recently returned from a trip to that country. In his paper he presented an account of the economic and industrial problems peculiar to the U.S.S.R. which must be considered in connection with its problems in radio. A description was given of the present laboratories and stations maintained there. Statistics and data concerning plans for future growth were presented.

The paper was discussed by Dr. Dellinger and a number of others of the fifty-four who were at the meeting. The informal dinner which preceded it was attended by twenty-one.

Personal Mention

W. E. Benham formerly with the International Telephone and Telegraph Laboratories has become a member of the staff of the Research Department of Marconi's Wireless Telegraph Company, Chelmsford, England.

G. J. Boerner of RCA Communications has been transferred from Bolinas, Calif., to Rocky Point, N. Y.

Lieutenant Commander C. W. Brewington, U.S.N., has been transferred from the Naval District Headquarters at San Francisco to the U.S.S. Louisville.

Previously with Hygrade Sylvania Corporation, Alfred Christeler

has become assistant to the chief of the radio and refrigerator department of Kaiser and Company of Berne, Switzerland.

L. E. Closson, previously with the Wurlitzer Manufacturing Company has become an engineer for Philco Radio and Television Corporation of Philadelphia, Pa.

Lieutenant L. R. Daspit, U.S.N., has been transferred from New London, Conn., to the U.S.S. S20 operating out of San Diego, Calif.

Formerly with the United Research Corporation, R. H. Dreisbach has become affiliated with the Electro Acoustic Products Company of Fort Wayne, Ind.

R. F. Durrant has been appointed United Kingdom Civil Aviation Signals representative at Heliopolis Aerodrome, Cairo, Egypt.

W. C. Evans, formerly manager of radio broadcasting for Westinghouse Electric and Manufacturing Company, has been transferred to Chicopee Falls as manager of radio.

Previously located in London, Harry Faulkner has become Assistant Superintending Engineer in the Post Office Engineering Department at Shrewsbury, Shropshire, England.

E. S. Fletcher, Communications Officer, U.S.C.G., has been transferred from Sault Ste Marie, Mich., to Cleveland, Ohio.

Captain R. A. H. Galbraith has been transferred from Woolwich, England, to the Office of the Director of Signals at Ottawa, Ont., Canada.

Dr. Alfred N. Goldsmith, formerly vice president and general engineer of the Radio Corporation of America, has established a consulting practice at 444 Madison Ave., New York City.

S. D. Gregory has been transferred from the East Pittsburgh plant of the Westinghouse Electric and Manufacturing Company to Chicopee Falls, Mass., where he has become assistant to the manager of the radio department.

Formerly with the British Thompson Houston Company, R. C. Hall has become professor of electrical engineering at the Crystal Palace School of Practical Engineering, London, England.

G. L. Haller, formerly chief engineer of E. A. Myers and Sons, has become an engineer for Amplivox Sound Laboratories, State College, Pa.

J. F. Hildebrand, formerly at Columbia University, has become business manager for Hildebrand, Lee, Parsons, and Price, consultants in New York City.

J. W. Horton, previously chief engineer of General Radio Company, has become a research associate in the department of electrical engineering at the Massachusetts Institute of Technology.

Captain R. C. Instrall has been transferred from Palestine to Catterick Camp, Yorkshire, England.

Previously with the Museum of Science and Industry, E. Kohler, Jr. has joined the research department of the Grigsby-Grunow Company in Chicago.

Formerly chief engineer of Arcturus Tube Company, W. L. Krahl has joined the staff of Hygrade Sylvania Corporation in Salem, Mass.

Previously with J. Dyson Company, Edward Lawrence has become a sound engineer for Faumont British at London, England.

F. J. Macedo is now radio engineer for the Radio Corporation (N.Z.) Ltd. at Wellington, New Zealand, having formerly been associated with W. Marko.

P. De Forrest McKeel has been transferred from East Pittsburgh to the radio broadcasting department of the Westinghouse Electric and Manufacturing Company, Chicopee Falls, Mass.

Formerly with the Stewart-Warner Corporation, B. B. Minnium has become chief engineer of the Erie Resistor Corporation, Erie, Pa.

G. K. Morrison, previously with the Federal Telegraph Company, has become a member of the research and development section of RCA Radiotron Company, Harrison, N. J.

G. B. Myers, Lieutenant, U.S.N., has been transferred from Newport, R. I. to the U.S.S. Chicago.

G. M. Neely, Lieutenant, U.S.N., formerly on the U.S.S. Northampton is now at Naval Communications, Washington, D. C.

Previously on the U.S.S. Milwaukee, L. W. Nuesse has been transferred to the U.S.S. Concord.

A. W. Peterson, Lieutenant, U.S.N., has been transferred from the U.S.S. California to the U.S.S. West Virginia.

Naval Research Laboratory at Bellevue, Anacostia, D. C., is now the location of J. J. Pierrepont, Lieutenant, U.S.N. who was formerly on the U.S.S. Concord.

Previously at Broadcast Station KYW, E. L. Plotts has joined the engineering staff of Columbia Broadcasting System in Chicago, Ill.

M. O. Sharpe has been appointed chief engineer of the Knoxville Police Radio System Station, WPFO.

Formerly with the General Electric Company, E. E. Spitzer has joined the research and development laboratory of RCA Radiotron Company at Harrison, N. J.

R. A. Stolle, formerly with Jackson-Bell Company has become engineer in charge of the Tomie Tucker Radio Corporation, Los Angeles, Calif.

Previously with the Westinghouse Electric and Manufacturing

Company, E. M. Stuckert has joined the staff of the Hygrade Sylvania Corporation at Salem, Mass.

Formerly with the Federal Telegraph Company, W. G. Wagener has now joined the staff of RCA Radiotron Company of Harrison, N. J.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

TECHNICAL PAPERS

A THEORY OF AVAILABLE OUTPUT AND OPTIMUM OPERATING CONDITIONS FOR TRIODE VALVES*

By
M. V. Callendar
(Research Department, Lissen, Ltd., Isleworth, England)

Summary—*The output characteristics of triode and pentode valves have been investigated in the past by many workers, using either the static method of plotting a whole family of curves, or the dynamic method of measurement with a harmonic analyzer: these have not, however, led to many simplifying generalizations, and are cumbersome and difficult methods if they are to be applied to each individual valve. In this paper, the form of the triode curves is first investigated experimentally, and the allowable limits of dynamic swing thus determined for any given per cent harmonic: on this basis, a series of expressions are mathematically developed giving the required output characteristics in terms of an easily obtained valve constant (i.e., the valve alternating-current resistance at* $V_a = 100$, $V_g = 0$; *this is the standard figure quoted by English manufacturers). Curves are given for output and correct plate current with various values of resistive load and of per cent harmonic limit, with or without conditions of limited anode dissipation. The case of the practical load with low direct-current resistance is examined, and the results are checked with a harmonic analyzer; the paper concludes with a few practical rules. It is hoped in a future paper to extend the analysis to cover push-pull circuits, the pentode valve, and the practical inductive load.*

A. Introduction

The Problem

Although much work has been done on the estimation of optimum operating conditions and available output from power valves, very few investigators seem to have been able to express their results in any generalized form; i.e., a form which can be extended to cover many cases without requiring complete repetition of experimental work on each new case. Of these, B. C. Brain[1] has produced the neatest theory; his work, however, involves two assumptions which are not realized in practice; viz., he assumes that all valve curves follow the three-halves power law, the amplification factor being constant throughout, while he also assumes a negligible value for I_{min},

* Decimal classification: R333. Original manuscript received by the Institute, December 11, 1931; revised manuscript received by the Institute, October 5, 1932; revised manuscript received by the Institute, January 11, 1933.

[1] *Experimental Wireless and Wireless Engineer*, March, (1929).

leading to an unknown per cent harmonic which will generally be above the permissible limit of 5 or 10 per cent: in addition, his formulas are not simple to apply and do not cover pentodes or many of the common practical cases (e.g., choke-fed load). Of the rest, several German writers[2] have admittedly considered some of these special practical cases but they have produced few really quantitative results. It, therefore, appeared of some importance to discover whether it was possible to arrive at simple working formulas for the required output constants and to attempt also to make these formulas readily correctable for the very many special cases found in practice.

The correct operating conditions for the pentode have only been partially understood, and that quite recently[3]; again, the output characteristics as generally specified with a fixed resistive load are not necessarily indicative of the practical performance with an inductive loud speaker; finally, there is the possible increase in output obtainable from the various varieties of push-pull circuits, and/or the use of a driving circuit designed to allow the output valve to run into grid current, but these last problems have unavoidably been left for consideration in a subsequent paper.

A List of the More Important Symbols Employed

$\left.\begin{matrix} I_a \\ V_a \\ V_g \end{matrix}\right\}$ are general terms for anode and grid currents and voltages.

E_a is direct-current potential on anode.

E_g is grid bias voltage.

V' is the value of V_g at which grid current starts (say, reaches 1 microampere under the standard static condition $V_a = 100$)

I_0 is value of I_a for triodes at $V_a = E_a$ and $V_g = V'$

I'_0 is value of I_0 at $V_a = 100$.

I_p is value of I_a at working E_a and E_g (with no alternating-current input).

I_{pw} is the value of working I_a at full alternating-current input.

I_{min} and I_{max} are minimum and maximum values of I_a on the working characteristic.

ρ_p is value of valve alternating-current resistance (impedance) at $I_a = I_p$ and $V_a = E_a$.

ρ' is value of valve alternating-current resistance at $V_a = 100$, and

[2] Von Ardenne, *Experimental Wireless and Wireless Engineer*, February and October, (1929); Forstmann, *Zeit. Hochfrequenz Tech.*, December, (1928); February, (1929); March, (1930). Also others in Proc. I.R.E., and *E.N.T.*

[3] Ballantine and Cobb, Proc. I. R. E., March, (1930); and Glessner, Proc. I. R. E., August, (1931).

$V_g = 0$; i.e. under the conditions taken as standard by English manufacturers.

ρ_m is value of valve alternating-current resistance for triodes at $I_a = I_{max}$.

μ is amplification factor.

R is the alternating-current resistance of the load.

R_0 is the direct-current resistance of the load.

W is the alternating-current power output to the load.

B. Preliminary Examination of the Valve Characteristics

The *form of the valve characteristic* must first be investigated. For this purpose, a large number of different valves were taken and their static $V_g - I_a$ and $V_a - I_a$ characteristics accurately plotted both upon ordinary graph paper and also upon log-log graph paper from which the power laws can be more easily seen. The variation of μ and ρ was also investigated independently on the usual type of dynamic bridge for valve constants. It was found that:

I. The form of the $V_a - I_a$ curve for $V_g = 0$ appeared always to approximate closely to a three-halves power law; on closer inspection of the characteristics of about 50 valves with the usual oxide-coated cathode or filament, three types of deviation from the ideal were observed:

(a) the curve is not of exactly three-halves power law form at very low anode voltage (say < 30 volts) owing to volts drop along filament, initial velocity of electrons, etc.

(b) the foot of the curve may be appreciably displaced relative to the $V_a = 0$ point owing to grid contact potentials.

(c) the curve may flatten out at very high currents owing to incipient filament saturation.

If now, we deal with the $V_a - I_a$ curve for $V_g = V'$, where V' is the point at which grid current starts (say reaches 1 microampere), the second of the above deviations is eliminated: again, in the case of the actual dynamic swing the saturation effects of the filament emission are much reduced, and results taken with a time-switch device for instantaneous measurements indicate that there is little likelihood of this third deviation occurring with valves of reputable manufacture during their normal life.

Third, as regards values of V_a above, say, 30 volts, the effects of (a) appear only as an addition to V'; thus, V' is usually about $+0.5$ to

+1.5 volts for 4- or 6-volt filament valves, as compared with −1 to +0.5 volt for those with equipotential cathodes.

In fact, we may usefully employ the equation $I_a = KV_a^{3/2}$ for the $V_a - I_a$ curve for $V_g = V'$: in particular, we may calculate the value I_0 at $V_a = E_a$ from the value I_0' at $V_a = 100$, using the equation $I_0 = I_0'/1000 \cdot E_a^{3/2}$; the accuracy appears to be of the order of ±3 per cent for values of V_a between 30 and 200 volts under the usual dynamic conditions (i.e., where deviation (c) is not appreciable).

Finally, we can calculate I_0 more approximately from the standard valve alternating-current resistance (ρ') as measured at $V_a = 100$ and $V_g = 0$, from the equation $I_0 = E_a^{3/2}/15\rho'$, which is obtained directly by differentiating the equation for I_a above. Here, however, we are neglecting V', and this will clearly introduce an error of $\mu/\rho' \times V'$ in I_0' which gives a percentage error of $\mu V' \times 1.5$, where V' may lie between −1 and +1.5 volts.

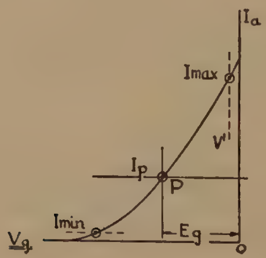

Fig. 1—Grid potential—anode current characteristic curve.

II. Even for "normal" valves, however, all the $V_g - I_a$ curves, and the $V_a - I_a$ curves for appreciably negative values of V_g, approximated more closely to a parabolic than a three-halves power law; any given curve could not be at all accurately represented by one simple equation of the form $I_a = kV^n$, the effective power (n) tending to become higher (e.g., up to cube law) both at low currents and at large negative biases.

III. Only one class of valve was found to deviate materially from the "normal" shape of II; this was representative of a particular make of indirectly heated valves, and gave very curved characteristics. It was found that this was due to a form of uncontrolled emission, apparently caused by insufficient spacing between cathode and grid relative to that between the grid wires: this has the effect of causing large variations in μ over the working range (compare with variable-mu tetrodes), one typical example giving $\mu = 16$ at $V_a = 50$ and $V_g = 0$, but falling to $\mu = 9$ at $V_a = 150$ and $I_a = 2$ milliamperes, these values showing the probable variation over the working swing. This variation

in μ does not, of course, appreciably affect the $V_g=0$ curve, which follows the equation given in I above: the variation is <10 per cent over the working range in "normal" valves, but some of these show greater variations than others.

The *allowable limits of dynamic swing* are next to be determined. (See Fig. 1.)

Further $V_g - I_a$ characteristics were taken with a resistance in series with the valve to represent the load; from these we can obtain the value of I_{min} for a given per cent harmonic by applying the usual formula for 2nd harmonic: per cent harmonic $=[(I_{max}+I_{min}-2Ip)/(2(I_{max}-I_{min})]\times 100$: this formula, strictly speaking, includes the per cents of 6th, 10th, etc., harmonics, but these may be considered negligible for our purposes.

From *theory*, we can only find that for any ratio I_{min}/I_{max} there will be a definite per cent harmonic in the case of parabolic or three-halves laws, which are the only types of curve easily amenable to calculation:

i.e., by expanding the expression for 2nd harmonic in terms of V_g we find that

if,

$$I_a \propto V_g^2, \text{ for } \frac{I_{min}}{I_{max}} = 0.45 \text{ we get } 5 \text{ per cent harmonic}$$

$$0.19 \text{ we get } 10 \text{ per cent harmonic}$$

$$0.00 \text{ we get } 25 \text{ per cent harmonic}$$

or if,

$$I_a \propto V_g^{3/2}, \text{ for } \frac{I_{min}}{I_{max}} = 0.30 \text{ we get } 5 \text{ per cent harmonic.}$$

These results are, of course, mainly of corroborative and academic interest.

From the *experimental* $V_g - I_a$ curves with load, we have derived corresponding curves for per cent harmonic against ratio I_{min}/I_{max} for a large range of different valves, anode volts and load ratios R/ρ_p: some of these are shown in Fig. 2, whence we conclude:

I. For the general run of valves (to be referred to as "normal"), even under widely different conditions, the ratio I_{min}/I_{max}, is remarkably uniform, average values being:

$$I_{min}/I_{max} = 1/6 \text{ for } 5 \text{ per cent harmonic}$$

$$= 1/20 \text{ for } 10 \text{ per cent harmonic}$$

$$= < 1/50 \text{ for } 20 \text{ per cent harmonic.}$$

There were two main directions of deviation from these average values:

(a) This ratio $I_{\min}/I_{\max}$ does not vary much for the 5 per cent harmonic limit over a wide range of $I_{\max}$: the values obtained for the 10 per cent harmonic limit, however, tended to indicate that $I_{\min}$ could be more accurately represented by an expression of the form $(1/K_1 \cdot I_{\max} + K_2)$: similar results held for 20 per cent harmonic, but $I_{\min}$ is so small in these cases that no appreciable errors will be introduced by neglecting this deviation.

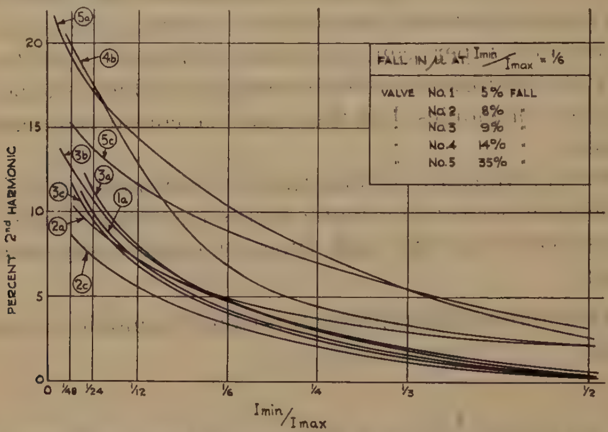

Fig. 2—Percentage harmonic from static characteristics with load.

Numbers 1, 2, and 3 are normal two-volt filament power type valves.
Number 4 is similar to the last, but is an individual example showing a "tail" to the characteristic.
Number 5 is a typical indirectly heated power valve.
Suffix: *a* refers to 2000-ohm load.
b refers to 5000-ohm load.
c refers to 12,000-ohm load.

(b) It has often been stated, even by usually trustworthy authorities, that a higher load ratio will give a notably straighter working characteristic: this idea, however, is evidently based upon the supposition that ρ alone varies, neglecting the variation in μ which latter will tend to increase the curvature of the dynamic characteristic as the load ratio increases: as a typical practical example, we may take curves 3a, 3b, and 3c in Fig. 2, which show an order of variation for normal working loads which is negligible for our purpose.

II. (a) As expected, however, the class of valves with variable-μ gave much higher values for $I_{\min}/I_{\max}$, (see curves 5a and 5c) even for $R \gg \rho_p$, and occasional cases of uncontrolled emission were also met with among other types, this being due to slight constructional defects. (Curve 4b.)

The *positive limit of grid swing* and hence the value of I_{max} will be determined by the rise of grid current to the point where the input impedance of the valve is seriously lowered, thus distorting the top of the dynamic swing. This question is a complicated one, and will be further examined both theoretically and practically in a subsequent paper: for the present, we shall assume, as has been done by most previous investigators, that no appreciable grid current is allowable; i.e., that $V_g = V'$ is the positive limit for grid swing.

From general principles it is clear that this V' limit for positive grid swing will only hold reasonably accurately in practice for cases where the previous stage has been designed with a view to obtaining the maximum amplification, thus presenting a high output impedance: Fig. 8 is a typical practical example, whence it is seen that an increase in output of only some 20 per cent may be expected unless considerable amplification is sacrificed.

C. Calculation of Formulas (Pure Resistive Load)

We shall assume only:

(a) That the $V_a - I_a$ curve for the V_g at which grid current starts (say, reaches 1 microampere) follows the law $I_a \propto V_a^{3/2}$: (where it is desired to simplify the formulas, we will further assume that $I_0' = 1000/15\rho'$.)

(b) That the value of I_{min}/I_{max} for the given per cent 2nd harmonic is an ascertainable constant for the valve, normally having the value given in the last section. (The 3rd harmonic may be neglected with triodes in comparison with the 2nd harmonic under all practical conditions except in push-pull working.)

(c) That the positive limit of grid swing is that value of V_g at which grid current starts (i.e., where $V_g = V'$).

These assumptions are partially justified by the last section, and are checked by the harmonic analyzer in Section E.

(d) In addition, we shall define the given anode voltage E_a as the direct-current potential actually on the anode, with no alternating-current input: thus, the total supply voltage is here $(E_a + RI_p)$. We shall consider first only a load having an alternating-current resistance equal to its direct-current resistance: the common practical case where the direct-current resistance is very low will be examined in the next section.

Case I. Anode current not limited by other considerations (e.g., anode dissipation not limited).

The problem is to find the correct anode current and maximum

output for the given valve at given E_a for any given load or per cent harmonic.

As a preliminary, we shall show that the ratio V_1/V_2 of the positive and negative anode potential peak excursions is determined only by the per cent 2nd harmonic in the output: for the purpose of this and the subsequent analysis we must consider the $V_a - I_a$ diagram, with load line drawn in, as shown in Fig. 3.

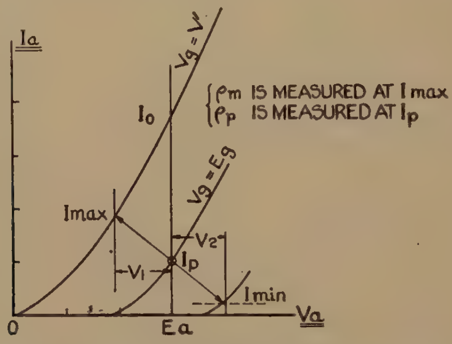

Fig. 3—Anode potential—anode current characteristic curve.

We have,

$$\text{per cent harmonic} = \frac{(I_{\max} - I_p) - (I_p - I_{\min})}{2(I_{\max} - I_{\min})}$$

$$= \frac{V_1 - V_2}{2(V_1 + V_2)}$$

$$= \frac{1}{1 + \dfrac{V_2}{V_1}} - \frac{1}{2}$$

Now, let,

$$\begin{cases} V_1 + V_2 = n_1 V_1 \\ I_{\max} - I_{\min} = n_2 I_{\max} \\ R = n_3 \rho_m, \text{ where } \rho_m \text{ is measured at } I_a = I_{\max}. \end{cases}$$

Then we have, from the equation for V_1/V_2 above, and the figures for $I_{\min}/I_{\max}$ for "normal" valves, the following values for the constants,

$$\begin{cases} \text{for } \ 5 \text{ per cent harmonic, } n_1 = 1.82,\ n_2 = 0.83 \\ \text{for } 10 \text{ per cent harmonic, } n_1 = 1.66,\ n_2 = 0.95 \\ \text{for } 20 \text{ per cent harmonic } n_1 = 1.43,\ n_2 = 1.0. \end{cases}$$

Then,

$$\begin{cases} I_{max} = (E_a - V_1)^{3/2} \times 1/15\rho' \text{ and } I_0 = E_a^{3/2}/15\rho' \\ V_1 + V_2 = R(I_{max} - I_{min}), \text{ or } n_1 V_1 = n_2 R I_{max} \\ \rho_m = \left(\dfrac{\partial V_a}{\partial I_a}\right)_{I_a = I_{max}} = \dfrac{10\rho'}{\sqrt{E_a - V_1}} \end{cases}$$

whence,

$$V_1 = \frac{n_2 n_3}{n_1} \rho_m I_{max}$$

$$= \frac{2n_2 n_3}{3n_1 + 2n_2 n_3} E_a$$

Similarly,

$$I_{max} = \left(\frac{3n_1}{3n_1 + 2n_2 n_3}\right)^{3/2} I_0.$$

Thus,

$$W_{max} = \frac{1}{8} n_1 V_1 \times n_2 I_{max} \text{ (since per cent 3rd harmonic is very small)}$$

$$= \frac{3^{3/2} n_1 \cdot {}^{5/2} n_2^2 \cdot n_3}{4(3n_1 + 2n_2 n_3)^{5/2}} \cdot E_a \cdot I_0,$$

and,

$$I_p = \frac{n_1 - n_2}{n_1} I_{max}$$

$$= (n_1 - n_2) n_1^{1/2} \left(\frac{3}{3n_1 + 2n_2 n_3}\right)^{3/2} I_0.$$

For convenience, we should express I_0 in terms of the easily measurable I_0' from the equation $I_0 = I_0' \cdot E_a^{3/2} \times 10^{-3}$: the values of the constants $C_1 = W_{max}/E_a^{5/2} I_0'$ and $C_2 = Ip/I_0' E_a^{3/2}$ have been plotted for a range of loads and for 5 per cent, 10 per cent, and 20 per cent 2nd harmonic, in Fig. 4: the calculated values have been confirmed in many cases by actual measurement on an ideal $V_a - I_a$ diagram.

To *simplify* the application of these curves we may always put $I_0' = 200/3\rho'$ provided we remember the additional errors discussed in Section B, which will generally not exceed ± 10 per cent.

Also, we can express ρ_m (the impedance at I_{max}) in terms of the more useful ρ_p (the impedance at I_p): we have:

$I_a \propto (V_a+\mu Vg)^{3/2}$ except at low currents:

hence $\rho \propto 1/I_a^{1/3}$ very nearly, over the range I_p to I_{max};

also, $I_p = n_1 - n_2/n_1 \cdot I_{max}$ from above, whence we obtain the factors noted on Fig. 4 for conversion to the more useful load ratio R/ρ_p. We can also deduce that ρ_p will lie between $1.5\rho'$ and $0.7\rho'$, for any value of E_a between 100 and 400, and thus use the standard impedance value in some cases where approximate results only are required (e.g., for $I_p = 1/4\, I_0$, $\rho_p = 16/\sqrt{E_a}\rho'$).

Conclusions for Case I. From the curves of Fig. 4, we see that, with normal valves, over the normal range of loads from ρ_p to $3\rho_p$, the avail-

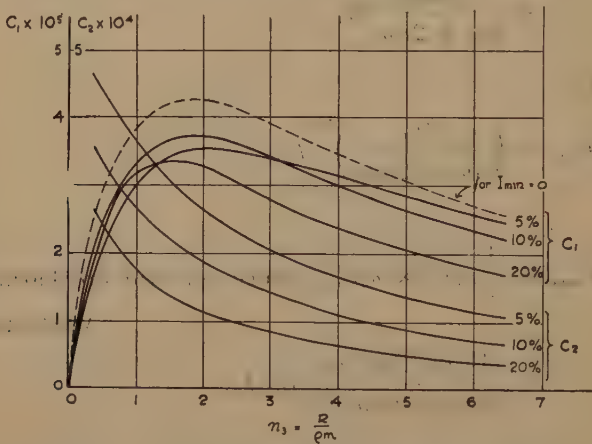

Fig. 4—Calculated curves for pure resistive load.

$W_{max} = C_1 I_0' E_a^{5/2}$ For 5 per cent harmonic, $\rho_m = 0.82\ \rho_p$

$I_p = C_2 I_0' E_a^{3/2}$ For 10 per cent harmonic, $\rho_m = 0.75\ \rho_p$

$I_0' = \frac{200}{3\rho'}$ approx. For 20 per cent harmonic, $\rho_m = 0.67\ \rho_p$

able output is always nearly $3.5 E_a^{5/2} I_0' \times 10^{-5}$ (to within about ± 10 per cent) for 5 or 10 per cent limit of harmonic, but that the correct plate current varies from 3.0 to $1.0 \times E_a^{3/2} I_0' \times 10^{-4}$, being smallest for large load and 10 per cent limit for harmonic. We also see that for loads above about $3\rho_p$, the available output actually decreases when operating conditions are set for a larger per cent harmonic than 5 per cent (i.e., when a lower plate current is used).

We can also estimate the corrections necessary to the above curves for abnormal valves. For one extreme we may take the alternating-current valves mentioned in Section B above: here the output will be uncertain and much smaller for 5 per cent harmonic, but will be only

some 20 per cent below normal for a 10 or 20 per cent limit for harmonic. For the other extreme, we may take the ideal case of $I_{min}=0$, when, for 5 per cent harmonic, we obtain an output some 20 per cent above normal as shown by the dotted line in Fig. 4, while the outputs for the higher per cent harmonic limits are almost unaltered.

Case II. Anode current limited by heating considerations, or regard for life of filament or economy of high tension power.

Here we require to find the maximum output from any valve to be run at given anode current and volts, and the correct load resistance to give this optimum output: this case will only have any practical meaning when the given anode current is less than the optimum I_p as calculated in Case I.

The required formulas can be obtained by merely rearranging the basic equations of Case I.

We have,

$$\begin{cases} n_1 V_1 = R(I_{max} - I_{min}) & \text{(limits } I_{min}) \\ I_{max} = (1 - V_1/E_a)^{3/2} I_0 & \text{(limits } I_{max}). \end{cases}$$

The first of these equations will limit the swing for low resistance loads, while the second will be in action for very high resistances, and thus only at a critical value of R (clearly R_{opt}) will the peak current reach both limits: hence we obtain:

$$R_{opt} = \left(\frac{n_1}{n_2} - 1\right)\left[1 - \left(\frac{n_1/n_2}{(n_1/n_2 - 1)} \cdot \frac{I_p}{I_0}\right)^{2/3}\right]\frac{E_a}{I_p}.$$

and then,

$$W_{max} = \frac{n_1^2}{8} \cdot \frac{1}{(n_1/n_2 - 1)}\left[1 - \left(\frac{n_1/n_2}{(n_1/n_2 - 1)} \cdot \frac{I_p}{I_0}\right)^{2/3}\right] E_a I_p.$$

The derivation of these formulas has not been set out in detail since they are not generally convenient for use in practice: however, we may obtain from them, or more easily from Fig. 4, a knowledge of the ratio of alternating-current watts output (W_{max}) to direct-current dissipation ($E_a\ I_p$) which is, of course, a factor of considerable practical importance. The variation of this factor with load and per cent harmonic has been plotted in Fig. 5, whence it is seen that the allowable per cent harmonic, while not greatly affecting the maximum available output, has a large effect upon this ratio of alternating-current output to direct-current dissipation. The advantages in this respect of a high resistance load (or low impedance valve) are very clearly brought out, but we

must note that there are practical disadvantages to such an arrangement, notably in the reduced sensitivity obtained.

The variation of output with load for a given fixed bias point (i.e., fixed I_p) may also be estimated from the equations given above. For loads less than the optimum, we have:

$$W_{\max} = \frac{R}{2}\left(\frac{I_{\max} - I_{\min}}{2}\right)^2 = \frac{R}{2}\left(\frac{n_1 n_2 I_p}{n_1 - n_2}\right)^2 \text{ from above;}$$

thus the output falls directly as R for a given per cent harmonic.

For loads greater than the optimum, where the swing is primarily limited by grid current, there is unfortunately no conveniently ex-

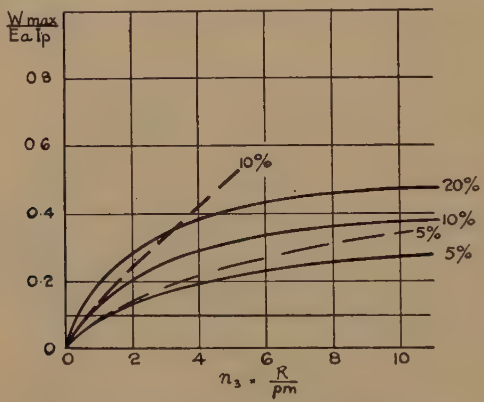

Fig. 5—Ratio of alternating-current output to direct-current power plotted against n_3 for pure resistive load

(Dashed lines for $\frac{R_0}{R} = 0$; see section D.)

pressible solution to the problem, since the per cent harmonic will become less as $I_{\min}$ rises with increase in R: we have $W_{\max} = (n_1 V_1)^2/8R$, whence we can say that, since V_1 increases slowly with R, the output will only fall off quite gradually. These results are of importance as indicating that, for the practical case of a load which varies with frequency, we should err if anything on the side of a higher load impedance than the optimum values obtained from the theory for a fixed load.

D. Theory for Load of Low Resistance to Direct Current

In actual practice, we shall generally be working with a transformer or choke fed load which will present a direct-current resistance R_0 low in comparison with its alternating-current resistance R: we shall at-

tempt, therefore, to evaluate the corrections to the formulas of the last section for the extreme case where R_0 is zero.

Now it is well known that, for the previously treated case where $R_0=R$, we have a rectified current I_r equal to the 2nd harmonic amplitude, and corresponding equal 2nd harmonic and rectified $(R\ I_r)$ voltages across the load. When, however, we have $R_0=0$, the rectified current will be increased, presumably in the ratio of the old total resistance (ρ_p+R) to the new resistance (ρ_p+R_0): the rectified voltage $(R_0\ I_r)$, on the other hand, must obviously fall to zero. Thus, when we sum the component peak voltages across the load for p per cent 2nd harmonic we see that the ratio of V_2 (=fundamental minus rectified current and 2nd harmonic) to V_1 (=sum of the three components) will

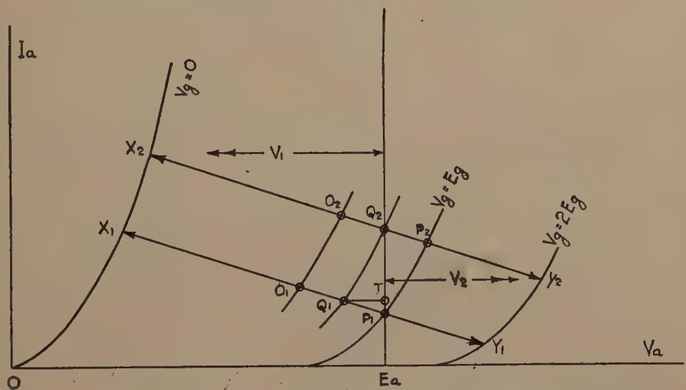

Fig. 6—V_a–I_a diagram for load of low direct-current resistance.

P_1 represents working anode current for no alternating-current input.
X_1Y_1 is working swing for $R=R_0$: O_1 is mid-point.
X_2Y_2 is working swing for $R_0=O$: O_2 is mid-point.
(For simplicity in drawing, V' has been assumed = zero, and the dissymmetry of the swing somewhat exaggerated.)

only be $1-p/1+p$ in the case of $R_0=0$ instead of $1-2p/1+2p$ as in the pure resistance case of Section C.

Further, if we follow Kilgour[4] and deal in terms of the working characteristic on the V_a-I_a diagram, we can confirm these results and visualize their application.

Thus in Fig. 6, the working characteristic for a pure resistance load would be X_1Y_1, the mean anode current and anode volts with full input on being represented by the point Q_1. When, however, we make $R_0=0$, it is clear from above that the corresponding point Q_2 must be on the

[4] "Graphical analysis of output performance," PROC. I. R. E., vol. 19, pp. 42–50; January, (1931). This paper should be consulted if any further explanation of the graphical method is desired.

$V_a = E_a$ line; hence, since O_2Q_2 must remain the same length for the same per cent 2nd harmonic, we must draw through Q_1 parallel to the $V_g=$const. line through O_1 to determine Q_2: thus we arrive at the new working characteristic X_2Y_2 by a simple construction[5] which assumes only that the $V_g=$const. lines are sensibly parallel over the short distances X_1X_2, Y_1Y_2, etc.: if they are not strictly parallel, we shall obtain a slight error in the per cent harmonic from the new characteristic, this being of the order $X_2Y_2 - X_1Y_1/X_2Y_2$, which is negligible for our purposes.

Here we note that the rectified current has increased from P_1T to P_1Q_2 (*i.e.*, in the ratio of one to $(1+R/\rho_p)$, as may be seen from the geometry of the triangles Q_1TQ_2 and Q_1TP_1). On the other hand, the difference $PX-PY(=V_1-V_2)$ between the two sides of the V_a swing is reduced to half: for values of R_0 which are appreciable but $<R$ an intermediate value of I_r and V_1-V_2 will evidently occur. If we turn, however, to consider the working V_g-I_a characteristic corresponding to Fig. 1, we see from Fig. 6 that the difference between the two sides of the working current swing about the point P_2 (on $V_g=E_g$) is not reduced, being still determined exactly as before by the per cent harmonic: thus we need no alteration to the previously determined values for $I_{\text{min}}/I_{\text{max}}$ or n_2.

Thus the formulas of Section C need modification in two definite ways:

I. We must insert different values for n_1 $(=1+V_2/V_1)$ in the final equations for W_{max} and I_p:

for 5 per cent harmonic n_1 will be 1.91 instead of 1.82
for 10 per cent harmonic n_1 will be 1.82 instead of 1.67
for 20 per cent harmonic n_1 will be 1.67 instead of 1.43

II. Since our calculations are made with respect to the actual working characteristic X_2Y_2, the value of I_p calculated from the formulas given in Section C will in this case be the correct working value I_{pw} with full input on. To obtain the standing current I_p for no input, therefore, we must subtract the rectified current (I_r) from above:

$$I_r = \text{2nd harmonic amplitude} \times (1 + R/\rho_p)$$
$$= n_2 I_{\text{max}}.p.(1 + R/\rho_p) \text{ for } p \text{ per cent 2nd harmonic.}$$

Here n_2I_{max}, the fundamental output, has been obtained during the calculation of W_{max}. The only difficulty here will occur in the relatively

[5] See also a letter from Bedford in *Wireless Engineer and Experimental Wireless*, November, (1931).

unimportant case where the largest per cent harmonic is allowed with a high load ratio: the value of ρ_p to be inserted in the formula for I_r will become appreciably different from that measured at I_{pw}, and, since the rectified current is here a large percentage of I_{pw}, the value for I_p will be liable to corresponding errors.

The new values for $W_{\max}$ and I_p have been plotted in Fig. 7: it is seen that the only material differences from the curves of Fig. 4 consist of a slightly larger output (order of 10 per cent) and a considerably

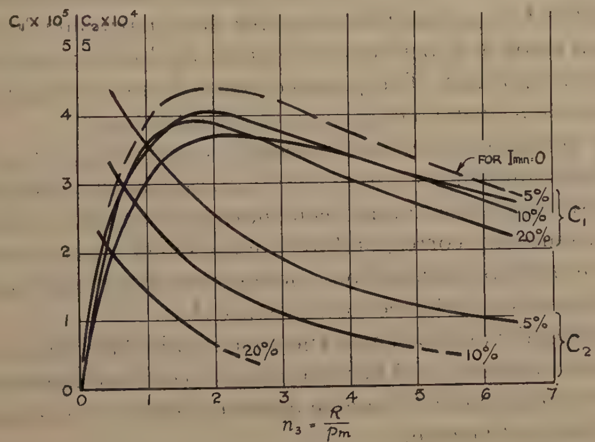

Fig. 7—Calculated curves for load of low direct-current resistance $\left(\frac{R_0}{R}=0\right)$

$W_{\max}=C_1 I_0' E_a^{5/2}$ For 5 per cent harmonic, $\rho_m=0.83\ \rho_p$

$I_p=C_2 I_0' E_a^{3/2}$ For 10 per cent harmonic, $\rho_m=0.78\rho_p$

$I_0'=\frac{200}{3\rho'}$ approx. For 20 per cent harmonic, $\rho_m=0.74\ \rho_p$

smaller plate current at the higher per cent harmonic limits: this latter difference is shown up more clearly by the dashed curves in Fig. 5, but the practical economy will not be quite as great as that apparent, owing to the larger rectified current when a transmission is being received. The effects of abnormal values of $I_{\min}/I_{\max}$ will be similar to those calculated for the pure resistive load, and, in general, the results of the two cases differ so little from a practical point of view that it has not been thought worth while to evaluate any numerical figures for examples where $0 \ll R_0 \ll R$, since both $W_{\max}$ and I_p will be intermediate between the values calculated for the limiting cases.

E. Limits of Accuracy of Formulas, and Check Measurements with a Harmonic Analyzer

If we leave out of consideration those practical cases in which an appreciable grid current is allowable, or where the load impedance is inductive in character, there are only two definite *sources of error* in the values of output for "normal" valves calculated from the formulas of the last two sections, and these correspond to assumptions (a) and (b) specified at the beginning of the section:

(a) Any percentage error p in the value of I_{max} calculated from a measured I_0' or ρ' will give an equal error p per cent in the output W_{max} (const.$\times E_a\, I_{max}$) at the given per cent harmonic limit: it will also give an error of p per cent in correct I_p, but this will clearly be relatively unimportant. The possible errors in measuring load resistance in terms of ρ_p instead of ρ_m may be included here.

(b) Any deviation from the "normal" value of I_{min}/I_{max} assumed for the calculation will, as can be seen from an inspection of the results of the last section, alter the per cent harmonic and the correct anode current without any correspondingly large effect upon the available output. The probable errors from these causes may be estimated from the previous sections, whence we may state that:

Any "normal" valve will give the output calculated from the measured ρ' to within about ± 15 per cent at a per cent 2nd harmonic within ± 1 of the figure given: if I_0' can be measured direct, the output will be given to within ± 10 per cent.

These errors have been further checked by means of *measurements with a harmonic analyzer*. Two tables of such results are given, together with values calculated from Fig. 6: the valves are numbered to correspond with the curves of Fig. 2. In the experiments, a choke of 200 henrys inductance and 1000 ohms resistance was used to feed the load; the alternating-current input from a low impedance source was always adjusted to give one microampere of grid current, whence the anode current to give 5 or 10 per cent harmonic could be quickly found, and

TABLE I

Measurements on various valves at load of approximately $2\rho_p$ ($E_a = 120$ volts).

Valve No.	Load	I'_0	Measured by analyzer: I_p (ma) 5%	I_p (ma) 10%	W_{max} (watts) 5%	W_{max} (watts) 10%	Calculated for "normal" valves: I_p 5%	I_p 10%	W_{max} 5%	W_{max} 10%
0	4,000ω	41.0 ma	13.3	9.0	0.247	0.277	14.0	8.75	0.240	0.260
1	5,000ω	38.5 "	10.9	7.7	0.234	0.256	12.4	8.1	0.226	0.247
2	4,000ω	45.0 "	14.4	9.3	0.252	0.277	14.6	9.6	0.264	0.289
3	10,000ω	20.0 "	6.6	4.3	0.116	0.130	6.2	3.75	0.117	0.128
4	10,000ω	27.0 "	8.6	5.2	0.127	0.146	6.9	4.25	0.158	0.164
5	5,200ω	40.7 "	26.8	12.8	0.100	0.229	11.1	7.7	0.234	0.257

TABLE II

Valve No. 2 on $E_a = 120$ volts

Load	Measured						Calculated					
	W_{max} (watts)			$O_{pt.}I_p$ (ma)			W_{max}			$O_{pt.}I_p$		
	5%	10%	20%	5%	10%	20%	5%	10%	20%	5%	10%	20%
2,000ω ($0.85\rho_p$–$1.2\rho_p$)	0.220	0.265	0.255	19.8	13.5	8.8	0.228	0.286	0.271	19.4	14.5	8.6
4,000ω ($1.8\rho_p$)	0.255	—	—	14.0	—	—	0.264	—	—	14.5	—	—
5,000ω ($1.45\rho_p$–$2.2\rho_p$)	0.260	0.276	0.260	12.6	8.6	4.2	0.264	0.282	0.274	13.0	7.8	3.7
6,000ω ($2.4\rho_p$)	0.263	—	—	11.0	—	—	0.260	—	—	11.5	—	—
8,000ω ($3.0\rho_p$)	0.241	—	—	9.0	—	—	0.246	—	—	9.2	—	—
0,000ω ($2.37\rho_p$–$3.6\rho_p$)	0.215	0.215	0.190	7.8	5.0	2.2	0.234	0.236	0.209	8.1	4.5	—

he corresponding fundamental output measured: subsidiary measure-nents determined I_0' and ρ_p for each case.

In Table I, valves Nos. 0–3 are "normal" by the I_{min}/I_{max} criterion, nd it is seen that the measured outputs and plate currents agree losely with the theoretical values. No. 4 exhibits a "tail" on its $V_g - I_a$ urves, and gives an output 10–20 per cent low as a result, while No. 5 s one of the class of indirectly heated valves with very curved charac-eristics, resulting in an output of the order of half normal (with very

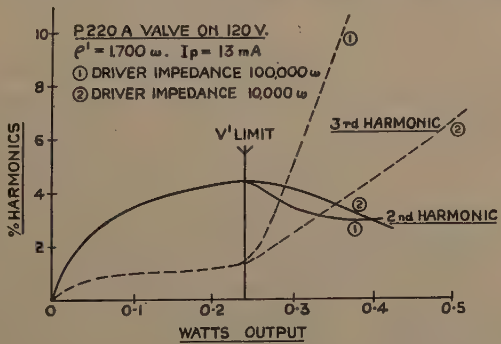

Fig. 8

high I_p) for 5 per cent harmonic, though even here we are only about 10 per cent low in output if 10 per cent harmonic can be allowed.

In Table II, we have the measured variation of W_{max} and I_p with load for a "normal" valve: the results agree well with theory except at the highest load, where the resistance of the choke will have a just appreciable effect.

The curves of Fig. 8 are appended as a typical practical example of the variation of harmonic with output for a given bias point: in particu-lar, the effects of grid current with a high impedance input circuit are clearly shown.

[*Note*: The harmonic analyzer was of the tuned circuit type, and a 50-cycle generator with filter was used for the alternating-current supply; the harmonic introduced by the supply or the feed choke was always $<$one-half per cent. The readings on the analyzer were accurate to approximately ± 2 per cent for the fundamental voltage and ± 5 per cent for the harmonic voltages.]

The only *large errors* in output calculated from the "normal" formulas are likely to occur in valves whose characteristics are unusually curved due to uncontrolled emission, or in cases where the valve has been rated more liberally than is warranted by its filament emission. (See Section B.) If the valve is suspected of either of these faults, we may most simply test for them as follows:

(a) measure the amplification factor under standard conditions ($V_g=0$, $V_a=100$) and repeat the measurement with bias added to reduce the anode current to one sixth of its previous value: the second value for μ should not be >12 per cent less than the first for a normal valve (see table on Fig. 2). Alternatively, a static $V_g=I_a$ curve (say at $V_a=100$) may be taken: the curvature of this should be such as to give <10 per cent harmonic by the usual rule for $I_{\min}/I_{\max}=1/6$.

(b) there should be no appreciable tendency for the emission to fall off when the valve is run for half a minute at 50 per cent in excess of the proposed working anode current.

There is one further limitation worthy of note, besides those mentioned in the first paragraph of this section: we have always given the output characteristics in terms of the actual direct voltage on the anode (E_a), and have tacitly assumed that the total supply potential applied to valve plus load did not vary with the current consumed. Now in the case where the amplifier is to be run from an alternating-current supply by means of a rectifier and smoothing unit, this latter condition will hold by no means exactly: thus we will here have a considerable increase in the effective direct-current resistance of the load. The output under these conditions can be estimated by comparing the cases of $R_0=R$ (Section C) and $R_0=$zero (Section D), whence we see that, provided the effective voltage dropping resistance of the power pack is not large compared with R (and it will not often be so in practice), we need only modify our output and plate current figures very slightly: moreover, even with exceptionally poor voltage regulation there will be no serious effect on the output at 5 per cent harmonic limit.

Apart from the above considerations, which apply to this particular theory, it is obvious that the accuracy of any measurements upon power output is limited by the partial instability of the filament emission: for instance the current rating of most valves is such that the

emission tends to "fade" if they are kept on for a minute at currents appreciably in excess of double their maximum rated working current, while some are not perfectly stable even at their normal rated current. From this point of view, the output characteristics calculated from the valve impedance at a reasonable current (if required, ρ' can be calculated from readings at an E_a of, say, 50 volts) are more likely to be accurate than those which depend for their accuracy upon the *exact* (i.e., to ± 1 per cent) stability of emission throughout the taking of a complete family of $V_a - I_a$ curves.

Finally, it must be remembered that one cannot detect by ear a change in acoustic power less than some 30 per cent, and similar considerations probably apply to figures for per cent harmonic; hence, it is more important to make formulas for output simple to apply and flexible to cover all cases than to attempt a great numerical exactness.

As regards the correlation of these formulas with the results of other investigators, we may note particularly that the output is here shown to be independent of the amplification factor of the valve, which latter has appeared explicitly in various other suggested formulas: for instance Brain[1] gives his output in terms of mu and another more recondite valve constant, but it can easily be shown that his formulas may be reduced, without any further assumptions, to the form const. $\times E_a^{5/2} \times I_0'$, this constant being generally somewhat (order of 20 per cent) larger than the corresponding factor C_1 from this paper owing to his assumptions as mentioned in the introduction. Again it is of interest to note that the standard impedance ρ', which has been often considered an unfortunate anachronism, may be conveniently used as a fundamental valve constant.

F. Practical Rules and Summary

In the previous sections we have worked out and checked what is, it is hoped, a logical theory, leading to values for the required output characteristics, but have not left the conclusions in a form necessarily suitable for practical radio engineering application.

To start with, it may have been noted with some surprise that, in the theory of Section C, no rules for *grid bias* (E_g) have been given. Nevertheless, if we neglect errors due to contact potentials, and volts drop along the filament, and also assume that the three-halves law holds for the $V_g - I_a$ curves down to a current $\doteq I_p$, we can obtain (using the constants of Section C) a formula for E_g of the form $E_g = f(n_1 n_2, n_3) E_a/\mu$. The expression is, however, complicated and not accurate enough to warrant its evaluation for all cases: instead, we have determined experimentally the bias required to give the normal correct

anode current (see below) $I_p = \frac{1}{4} I_0 = 1/4 \times 10^{-3} E_a^{3/2} I_0'$: on a test of about 30 valves of the usual oxide-coated filament types, we find that this correct bias for normal operating conditions is given by $-E_g = 0.6 E_a/\mu$ with a maximum likely error of approximately ± 2 volts, mainly due to contact potentials: such an error is, however, far more serious than any likely to occur in the formulas of Sections C and D,

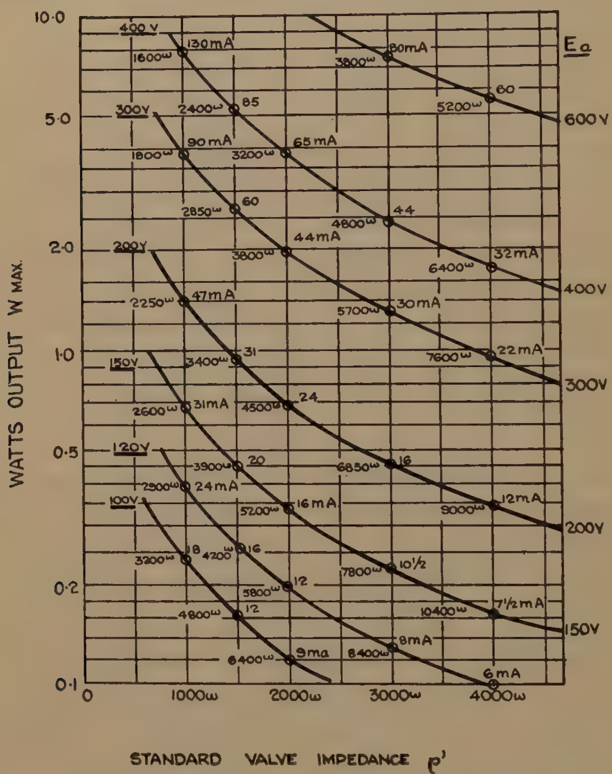

Fig. 9—Output characteristics for triodes.

For five per cent harmonic limit: at correct plate current and load as marked upon curves.

and conditions should therefore always be set by anode current where possible. It is of interest to note that the self-biasing resistance in this case should be approximately $= 36/\sqrt{E_a} \times \mu/\rho'$.

In any practical case, numerical values for output and plate current may be most conveniently obtained from the *curves of Fig. 9* in conjunction with the following notes on the same:

(a) The graph gives the maximum output $(E_a^{5/2}/400\rho')$ for 5 per cent harmonic limit and the optimum load $(2\rho_p)$ for a valve of any standard

impedance (ρ') on any anode voltage (E_a) *provided* the valve can be run on the optimum plate current ($E_a^{3/2}/60\rho'$) as marked on the curves. The correct bias for these conditions is approximately $0.6E_a/\mu$, but the working point is best set by I_p.

(b) If the plate current is limited for any reason to a value below the optimum marked on the curve, the output will not be greatly reduced provided we use a higher impedance load: *e.g.*, using three times the load marked on the curves, we shall obtain 0.8 of the marked output if we run on a plate current of half that marked.

(c) If we allow up to 10 per cent limit for harmonic, we may use a plate current of only two thirds of that marked, but the maximum available output will not be materially increased. If we do not require the maximum possible output or sensitivity from the valve, we may obtain the most distortionless reproduction by the use of a high plate current (say, one and one third of that marked) with a high impedance load (say, double normal).

The rules given here are directly obtainable from the curves of Section C with the exception of the optimum load, which has been selected as the best compromise taking into consideration the variable nature of the practical load, and the advantage of economy in plate current. The detailed discussion of those practical cases where the load is inductive and where some grid current may be allowed cannot be given here, but it may be stated that the above rules do not appear to need any material modification for average practical use, provided that the load is adjusted to present the optimum impedance at about 500 cycles.

For *valve specification* purposes, in any case, we must avoid those types of practical conditions where the output is as much characteristic of the circuit as of the valve, and thus the curves may be here used with every confidence.

Acknowledgment

In conclusion, the author desires to acknowledge his indebtedness to Messrs. Lissen, Limited, for permission to publish the results of work performed in their laboratory.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

DIODE DETECTION ANALYSIS*

By
C. E. Kilgour and J. M. Glessner
(Crosley Radio Corporation, Cincinnati, Ohio)

Summary—*This paper gives the current and output relations for the linear and square-law diode detector in terms of the detection efficiency and the detector and load resistances. The square-law case is shown to yield results similar to the linear for large input potentials and reasonable values of load. Employing the rectification-current—input-voltage characteristic, the graphical method is used to show that serious distortion is developed for deep modulation whenever an appreciable audio load is shunted across the diode resistor. It is found that $M=1+D(A-1)$, where M is the modulation ratio at which cut-off begins, D the detection efficiency, and A the ratio of the audio impedance to the direct-current value of the diode load. The value of M is shown for various load conditions in both the linear and square-law cases. The paper shows that the use of a small condenser across the load decreases the detection efficiency but increases the input impedance. The connection to the driving circuit of a second diode for the development of automatic volume control potential is shown to cause distortion when such a tube is used with a delay bias. Mention is made of the importance of considering the impedance of the driving circuit, and attention is called to the failure of any of the methods of analysis to consider all of the factors involved.*

THE diode detector has been much used in recent broadcast receiver design. Its popularity is due to several features prominent among which are its ability to stand up under any practical input without overloading, to supply ample voltage for automatic volume control, and to furnish a high quality audio output.

From the standpoint of fidelity, an ideal detector must produce a voltage that is a facsimile of the signal wave envelope. The so-called ideal linear detector may, under certain conditions, accomplish this transformation. This theoretical detector is characterized by an infinite resistance in the negative voltage range and a finite constant resistance in the positive range. This resistance is hereafter referred to as R_D. Such a device when used with a series resistance, hereafter called R_L, by-passed by a suitable condenser, may be shown to develop a unidirectional voltage across the resistance proportional to the amplitude of the impressed high-frequency voltage. Calling the developed voltage E_2 and the peak input voltage E_1 we may define the detection efficiency D as the ratio of E_2 to E_1 without regard to sign.

* Decimal classification: R332×R134. Original manuscript received by the Institute, January 4, 1933. Presented at the Fall Meeting of the Institute of Radio Engineers at Rochester, New York, November 15, 1932; and before the Cincinnati Section of the Institute of Radio Engineers, in part February 16, 1932, in entirety November 22, 1932.

Assuming that the condenser has zero impedance to the signal frequency, the following relations may be developed by standard mathematical procedure:

$$I = \frac{E_1}{\pi R_D}\left[\sqrt{1 - D^2} - D\cos^{-1} D\right] = \frac{E_2}{R_L} \tag{1}$$

$$\frac{R_L}{R_D} = \frac{\pi D}{\sqrt{1 - D^2} - D\cos^{-1} D} \tag{2}$$

While this last expression is not solved explicitly for the detection efficiency it is evident that D is independent of the input voltage and is determined entirely by the ratio of R_L to R_D. Assigning values to D and solving for R_L/R_D the relation may be plotted as shown in Fig. 1.

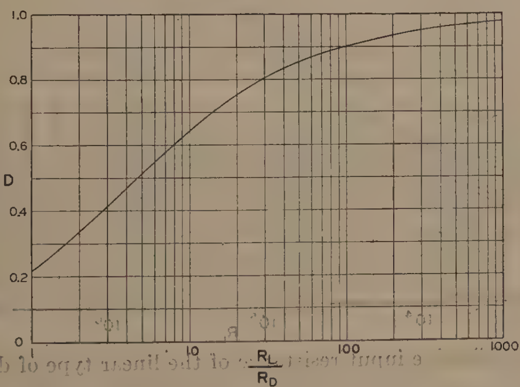

Fig. 1—Detection efficiency of the linear type of diode.

The effective input resistance of the linear detector circuit may be determined by considering the power delivered by the driver transformer, which is here assumed to be resonant to the input frequency. Integrating the product of the instantaneous current and voltage for one cycle and dividing by 2π yields

$$W = \frac{E_1^2\left[\cos^{-1} D - D\sqrt{1 - D^2}\right]}{2\pi R_D}. \tag{3}$$

This expression is equal to $E_1^2/(2R_E)$, where R_E is the effective input resistance, hence

$$R_E = \frac{\pi R_D}{\cos^{-1} D - D\sqrt{1 - D^2}} \tag{4}$$

Using different symbols, equivalent expressions were derived some years ago.[1] It will be noted that when D is zero, $R_E = 2R_D$, and when D is unity R_E is infinite. By making use of the relation between R_L/R_D and D as shown in the graph of Fig. 1, a plot of the effective resistance against R_L for various values of R_D may be made as in Fig. 2.

It will be noted that when R_L is large as compared to R_D that the effective resistance approaches one half of the load resistance in value. It is also apparent that the input resistance is independent of the applied voltage. The relation derived by Terman and Morgan,[2]

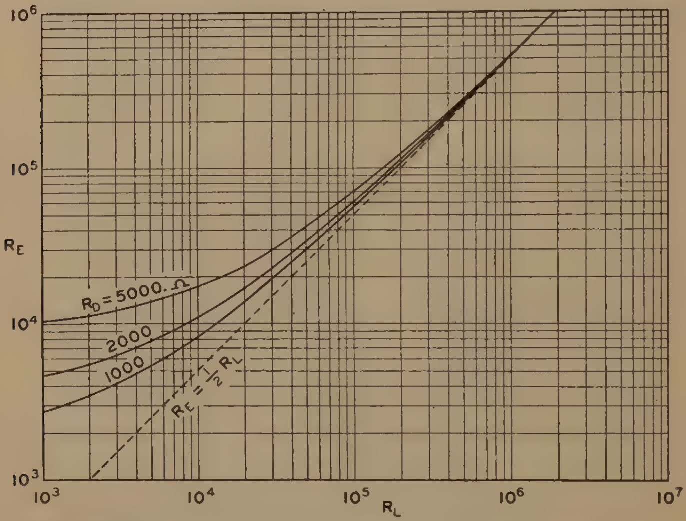

Fig. 2—Effective input resistance of the linear type of diode.

$R_E = R_L/2D$, will be found to yield quite similar results for values of D greater than 0.8.

It is true that no device has ideal linear detector characteristics although diodes may approach linearity for large inputs. However, for many diodes the current depends quite closely on the square of the voltage above the point where current starts, or designating this point as E_0,

$$i = \frac{(e - E_0)^2}{R_D}$$

where R_D is the inverse of the conductance per volt squared.

[1] F. M. Colebrook, "The theory of the straight line rectifier," *Experimental Wireless and Wireless Engineer*, vol. 7, no. 86, p. 595; November, (1930).

[2] F. E. Terman and N. R. Morgan, "Some properties of grid leak power detection," Proc. I.R.E., vol. 18, no. 12, p. 2160; December, (1930).

Defining the detection efficiency as

$$D = \frac{E_0 - E_2}{E_1} \tag{5}$$

ie same procedure as that followed in the linear case will develop the llowing relation:

$$\frac{R_L}{R_D} = \frac{2\pi\left(D - \frac{E_0}{E_1}\right)}{E_1[(1 + 2D^2)(\cos^{-1} D) - 3D\sqrt{1 - D^2}]}. \tag{6}$$

is evident in this case that the efficiency depends not only on $_L/R_D$ but on E_1 as well. The factor E_0/E_1 is unimportant when E_1 ıd D are large. In Fig. 3 D is plotted against R_L/R_D using E_1 as

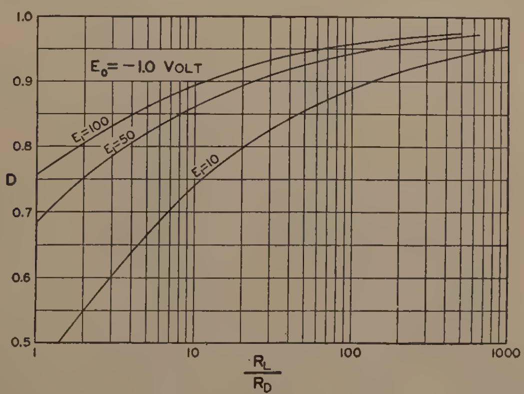

Fig. 3—Detection efficiency of the square-law type of diode.

parameter, and assuming a reasonable value of minus one volt for $_0$. It is to be noted that the efficiency varies considerably with input oltage for low values of R_L/R_D.

Proceeding as in the linear case, the power supplied by the resonant riving circuit is

$$W = \frac{E_1^3[\frac{2}{3}\sqrt{1 - D^2}(D^2 + 2) - 2D\cos^{-1} D]}{2\pi R_D} \tag{7}$$

quating this to $E_1^2/(2R_E)$,

$$R_E = \frac{\pi R_D}{E_1[\frac{2}{3}\sqrt{1 - D^2}(D^2 + 2) - 2D\cos^{-1} D]}. \tag{8}$$

Vhen D is zero $R_E = 3\pi R_D/(4E_1) = 2.34 R_D/E_1$ and when D is unity $_E = \infty$ as before. This expression becomes indeterminate for zero in-

put. However, when current starts with a negative voltage on the anode it is evident that for infinitesimal signals the input impedance is indicated by the slope of the static characteristic at its point of intersection with the load line. For the square-law case,

$$R_E = \frac{R_D R_L}{\sqrt{R_D^2 - 4R_D R_L E_0} - R_D}. \tag{9}$$

When $R_L \gg R_D$ and $E_0 = -1$,

$$R_E = \frac{\sqrt{R_D R_L}}{2}. \tag{10}$$

Making use of the graphs of D against R_L/R_D for various values of E_1, the relation between R_E and R_L can be shown as in Fig. 4. For

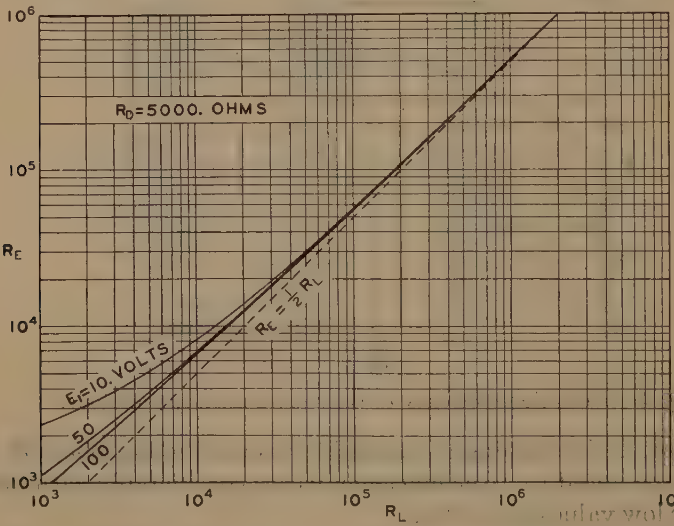

Fig. 4—Effective input resistance of the square-law type of diode.

very low values of R_L, R_E varies with the input voltage but for any load above 50,000 ohms the effective resistance is very close to one half of the load resistance.

From the preceding discussion it is evident that in the linear case with any particular load, the developed voltage bears a constant ratio to the input voltage. With square-law characteristics this is approximately true for reasonable values of load resistance. In both cases the effective resistance of the diode circuit is largely independent of input voltage. All this implies that the potential developed across the diode resistor will vary proportionally with the amplitude of the carrier en-

velope. Under the conditions of practical use, this proportionality may be considerably disturbed. Just how this may occur can probably be best explained by a graphical consideration of the diode's action.

In the case of a vacuum tube amplifier the familiar family of plate-current—plate-voltage curves with E_g as a parameter can be used to give a picture of the action resulting in the plate circuit when an alternating voltage is applied in the grid circuit. With the diode detector an equally useful family of characteristic curves can be drawn. Fig. 5 shows such a plot for an ideal linear detector. Assuming that the load capacity has negligible audio by-pass effect but infinite conductance at the carrier frequency, using the direct-current voltage across the grid resistor as the abscissa, the current through the diode for the

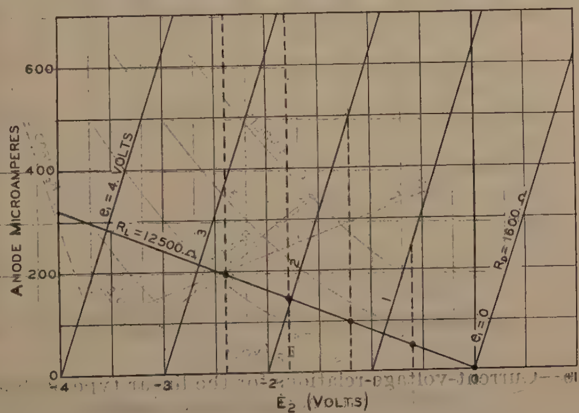

Fig. 5—Instantaneous current-voltage relations for the linear type of diode.

ordinate, and the instantaneous value of the input voltage as a parameter, a family of curves may be developed. It is obvious that these curves will be merely a repetition of the initial grid-current—grid-voltage curve, displaced to the left by an amount equal to the applied voltage. This, of course, would be true no matter what might be the form of the current-voltage curve. In the diagram shown the tube has a resistance to positive potentials of 1600 ohms. From such a set of curves it is possible to determine just what will happen with an input voltage of any wave form with any value of load in the circuit. Since this fictitious tube is to be operated without any bias other than that developed across the load resistor, the resistance may be represented by drawing a straight line starting from the origin with a slope such that for any point on the line, E/I equals the resistance value. In the diagram this has been taken as 12,500 ohms. Assuming a sinusoidal input of one peak volt and approximating the value of the developed

voltage, it is possible to draw in a line of action which is vertical be-cause of the zero load impedance to the signal frequency, lay out a graph of the current wave for one cycle, find the average value, and then check back to see whether or not this current will maintain the assumed voltage. If the first approximation is not correct, the line of action may be shifted and the current recalculated. The intersection of the determined line of action and the load line gives the rectified current for the specified input and resistance, or without regard to the load resistance determines the rectified current for a particular input and bias voltage. By a continuation of the procedure, the value of the rectified current for various input voltages may be determined.

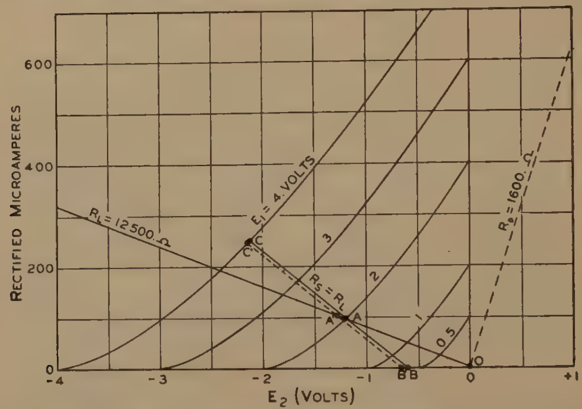

Fig. 6—Current-voltage relations for the linear type of diode with various alternating-current inputs.

Although it will be found that considerable skill will develop in making the approximations involved, still the process is rather labori-ous and whenever it is possible the rectified current may be determined experimentally with much greater ease. This is done by applying vari-ous alternating voltages and measuring the direct current flowing for various bias potentials. A new family of characteristics may now be developed as shown in Fig. 6. As before, the voltage across the resist-ance is the abscissa, but rectified current is the ordinate, and the peak value of the input voltage is the parameter. The tube is assumed to be the same linear detector taken for the preceding figure. In this ideal case the rectified current is indicated by equation (1) and the curves shown have been calculated accordingly. Note that although the static line is straight, the lines for rectified current are curved. However, the curvature is such that any line drawn through the origin is cut into equal portions by equally spaced E_1 lines, which, of course, must be the case since D was shown to be independent of E_1.

With the load line in place, it is at once apparent just what current and voltage are developed for any applied voltage within the range of the graph. Such a linear detector does a perfect job of developing direct current proportional to the input. However, in the detection of a modulated signal, the variation of the voltage across the load resistor is usually impressed on the following tube by some such coupling circuit as shown in Fig. 7. This coupling produces an audio shunt across the diode resistance without changing its value to direct current. Assuming that the coupling condenser is large, the audio impedance between the diode resistor terminals is determined by R_L and R_S in parallel. Assuming that $R_S = R_L$, the audio load will be $(1/2)R_L$.

If now a carrier having a peak value of two volts is applied, the current and voltage developed are indicated by point A. If this car-

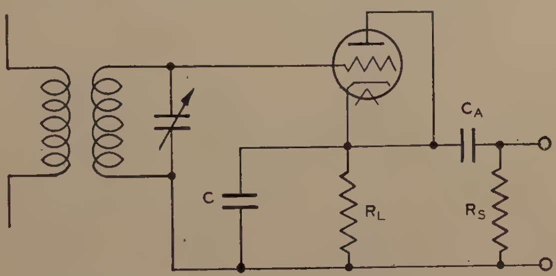

Fig. 7—Typical diode detector circuit.

rier is modulated at an audio rate, the line of action will not follow the direction AO but will follow a line whose slope is determined by the value of the audio load. As this is one half of the direct-current load, the action must follow the line CAB. This new line of action introduces no difficulty for low modulations except to reduce somewhat the developed voltage and produce slight nonlinearity for the line is no longer directed toward the origin and hence does not produce equal intercepts in crossing equally spaced input voltage lines. However, for deep modulation it is seen that serious distortion is developed. Before the carrier reaches zero the current is exhausted, and since current cannot flow in the reverse direction, the lower peaks of the developed audio wave are flattened. In the present example, cut-off is reached when the carrier has been reduced to approximately 0.6 of a volt, equivalent to 70 per cent modulation.

With 100 per cent modulation, the action in swinging from C to B will evidently cause an increase in the average direct current flowing through the load resistor, thus increasing the average value of the direct-current voltage across R_L.

To determine just where the new line of action will fall, a line such as $C'A'B'$ is assumed, and the direct current developed for one cycle of the carrier envelope is plotted. If the proper line has been chosen the average current for one cycle will be found to have the value indicated by the ordinate of the point A'. This movement of the line of action causes an increase in width of the flat portion of the audio wave. However, this secondary effect is negligible for ordinary values of the audio shunt. Since the average value of the direct current developed is practically independent of the audio shunt it can be said that the effective input resistance of the diode to the average carrier depends only on the direct-current value of the load. However, even for degrees of modulation lower than those producing cut-off, the audio

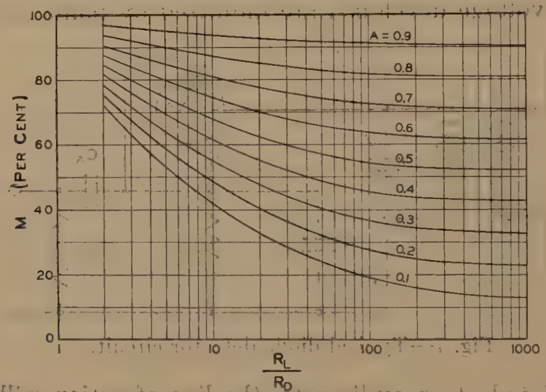

Fig. 8—Degree of modulation at which cut-off distortion appears with the linear type of diode. A is the ratio of audio to direct-current load impedance.

voltage is reduced so that its peak value is no longer equal to the product of the direct voltage and modulation factor.

From the geometry of the figure, it can be shown that the modulation at which cut-off distortion will start is

$$M = 1 + D(A - 1) \tag{11}$$

where A is the ratio of the audio to the direct-current load. Or

$$A = \frac{R_S}{R_L + R_S}. \tag{12}$$

Using the graph of the relation between D and R_L/R_D, Fig. 8 shows M against R_L/R_D for various values of A.

The rectified current characteristic for the square-law case, Fig. 9, is similar to the last but the curves trail off considerably more and it is

apparent that the action is not linear. This is particularly noticeable because the graph covers but a small voltage range and shows a low value of R_L. The dashed line near the bottom of the sheet is for a load of 250,000 ohms. The action is seen to be much more linear. Note that E_0, the potential at which current starts, has been chosen as minus one volt. This fairly common value has been used throughout wherever it is necessary to assign a definite value to this constant. The effect

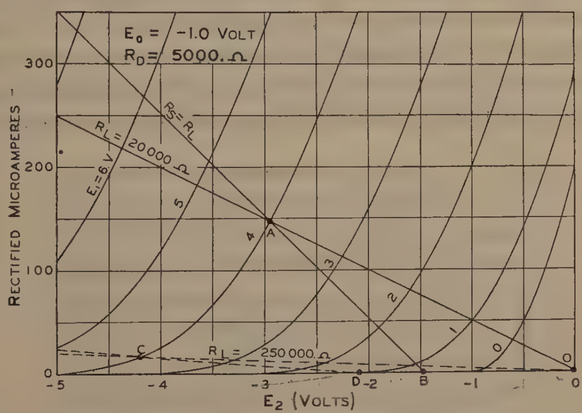

Fig. 9—Current-voltage relations for the square-law type of diode with various alternating-current inputs.

of the audio shunt to the load resistance is much the same as before. The critical modulation in this case is:

$$M = 1 - \frac{E_0}{E_1} + \frac{E_2}{E_1}(1 - A). \tag{13}$$

When E_0 can be neglected this reduces to the same expression found for the linear case. However, since D depends on E_1 as well as R_L/R_D, the plot of M against this last factor is somewhat different. Fig. 10 shows the relation for an input of 100 volts. For smaller values of input the cut-off distortion is lower for any particular value of A.

So far it has been assumed that the by-pass or grid condenser has zero impedance at the carrier frequency and zero admittance at the modulation frequency. Obviously such is not the case. The fact that the condenser exhibits impedance at the signal frequency complicates the problem considerably. To determine the effect of the condenser and resistor on the voltage applied to the diode terminals and hence on the efficiency and effective load it is necessary to solve for the instantaneous relations in the circuit. Because of the fact that the diode current is a discontinuous function of voltage, a straightforward solu-

tion is impossible. W. B. Lewis[3] has shown an ingenious method of arriving at the answer in the case of the linear diode. Assuming a sinusoidal input voltage, the expression for the voltage across the load for the interval during which the net voltage on the diode is positive is developed. Assuming an arbitrary value for the initial voltage across the load and using the solution for the instantaneous relations, the developed voltage is plotted for the period during which the detector is conductive. When the applied voltage falls below the developed potential, the graph is continued by plotting the exponentially falling voltage for the rest of the cycle, that is, until the potential across the diode is again positive. If the final voltage differs from the one assumed

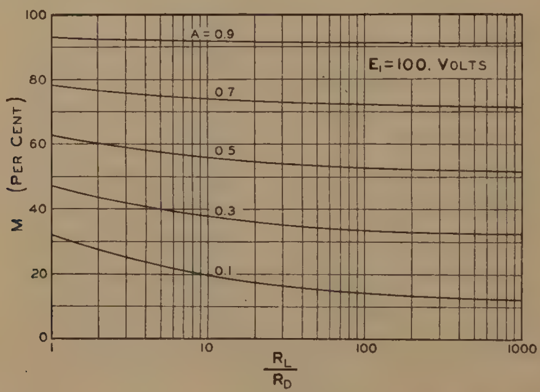

Fig. 10—Degree of modulation at which cut-off distortion appears with the square-law type of diode. *A* is the ratio of audio to direct-current load impedance.

another trial is made. By this method then a graph of the voltage developed across the load is obtained for a cycle of the applied high-frequency signal, and the value of the direct-current component and the amplitude of the high-frequency voltage may be found. The results indicate that with moderate values of R_L/R_D and with a condenser of fairly small capacity, there may be a sizable high-frequency component across the load. Since this voltage reduces that applied to the diode, the size of the condenser has a bearing on the efficiency and the effective load.

Lewis calculates that with R_L/R_D equal to four the efficiency will vary from 0.25 to 0.47 as the condenser is increased from zero to infinity in value. It may be shown for the linear case with no condenser that

[3] W. B. Lewis, "The detector," *Wireless Engineer and Experimental Wireless*, vol. 9, no. 108, p. 487; September, (1932).

$$D = \frac{1}{\pi} \times \frac{R_L}{R_L + R_D} \tag{14}$$

$$R_E = 2(R_L + R_D). \tag{15}$$

The square-law case is difficult if not impossible of solution. However, for large input voltages and large values of R_L the relations are apparently the same as for the linear case.

As the capacity is increased from zero, the factors mentioned will vary from the values indicated in (14) and (15) to those shown on the graphs where infinite capacity has been assumed. It is impossible in practice to realize the condition of no capacity. If no condenser is used across the load, there will still be a certain amount of circuit capacity. In this case the input capacity of the diode itself may be of the same order as the capacity across the load resistance, hence the high-frequency current through the system may reverse on the negative half of the cycle, thus lowering the rectification efficiency and producing a large high-frequency component across the load resistance.

Although the size of the condenser has a bearing on the detector efficiency, it is very frequently true that the use of a condenser of fair size is due to the need of reducing the high-frequency voltage fed into the audio amplifier. The wave form of this high-frequency voltage is far from sinusoidal indicating the presence of many harmonics whose suppression may be quite important. It is possible to use a small condenser and choke in a resonant circuit to take care of the carrier frequency but such an arrangement is obviously ineffective at the harmonic frequencies. This consideration may lead to the use of a capacity of such size as to have appreciable conductance at audio frequencies. This produces not only the same difficulties as does a capacity coupled resistive shunt but because of its reactive nature causes the line of action to become an ellipse. With heavy modulation the audio wave is not chopped off squarely, as was the case with the resistive shunt, but is sliced off diagonally.

This shunting effect of the condenser may be considerably lessened by the fact that the high audio frequencies at which the shunt is most effective are usually of less amplitude than the low frequencies so that with the customary composite signal the line of action will not be a large ellipse but will in the main follow the resistive load line about which it will describe an elliptical spiral.[4]

Sometimes, in order to develop a delayed automatic volume control voltage, a second diode plate shunted by a resistance is coupled through a condenser to the detector producing the signal. A voltage

[4] Unpublished work of Charles Travis, R.C.A. License Engineering Service.

source is placed in the circuit of this diode to delay the development of direct current across the resistance. Up to the time when this second anode becomes conductive, the arrangement acts as an ordinary shunt, first to the driving circuit at high frequency and second to the detector load at audio frequency. When current starts to flow, the radio-frequency shunt is increased in conductance, and since current may flow during but part of the envelope cycle, audio distortion may be developed because of the varying load on the driving circuit. These difficulties will be minimized by the use of a relatively high resistance in the shunt circuit. It is evident that the value of this resistance determines not only the direct shunt impedances but also the value of the effective diode resistance. It is to be noted that this automatic volume control diode circuit causes distortion only because of the delay voltage. If the two detector circuits are so arranged as to develop the same voltage, the second will not act as an audio shunt but only as a substantially constant radio-frequency shunt to the driver circuit.

For the greater part of this discussion it has been assumed that the driving generator has zero impedance. As the diode is commonly fed by a tuned transformer, this is of course not usually the case.

Calling the driver impedance Z_2 and the driver voltage E_g we have the following relation.

$$E_1 = E_g \frac{R_E}{R_E + Z_2}.$$

Travis[4] has used the graphical method to show the effect of the driver impedance.

For the linear case and also approximately for the square-law case the effective diode resistance depends only on D, which in turn is directly determined by the value of load resistance. Thus when the line of action is along the direct-current load line, E_1 will be directly proportional to E_g. However, with an audio shunt, R_E varies during the audio cycle, falling in value as the voltage swings above average and increasing as the voltage falls. As a consequence, unless R_E is large as compared to Z_2 the modulation reduction mentioned earlier will become more serious.

When the generator impedance is considered, it is evident that not only is the audio voltage produced less than the product of the direct current developed and the modulation factor, but actually the carrier voltage supplied by the driver circuit is reduced in modulation.

If the driving circuit is regarded as part of the detector system, that is if the direct current developed across the diode resistance is compared with the amplitude of the high-frequency applied to the

driver tube, the gain or conversion factor may be almost independent of the size of the load condenser for some combinations of coupling transformers and diode loads. With a small condenser, the detection efficiency is low but the input impedance is high so that the gain in the tuned circuit is high. With a large condenser the detection efficiency is higher, but because of the lower input impedance the voltage across the tuned circuit is less than with the smaller condenser.

In considering the action of a diode detector in a receiver it is essential that the driver tube and coupling transformer be considered with the detector and its load as a single system, for the voltage applied to the detector must be approximately a linear function of the input voltage to the driver tube if distortionless performance is to be expected. Ideally, the driver tube must be so biased and loaded that it will act linearly from zero input to twice that of the maximum value of the average carrier at which the receiver is designed to operate.

In concluding it was considered desirable to mention limitations in the various methods available for analyzing diode detector action. The equations given are restricted by certain ideal assumptions. The graphical system as used does not take into account the effect of by-pass condenser reactance. Conversely, while Lewis can handle this reactance, his method leads to complicated cubic equations when the generator impedance is included. It should also be remembered that he handles the linear case exclusively.

Each line of attack has shown difficulties which appear in the practical application of the diode and has suggested precautions which must be taken to obtain high quality performance with all degrees of modulation.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

APPLICATION OF TRANSFORMER COUPLED MODULATORS*

By
J. A. Hutcheson
(Westinghouse Electric and Manufacturing Company, Chicopee Falls, Mass.)

Summary—*A brief résumé of Heising's modulator theory is presented together with a discussion of the class B operation of tubes in a push-pull audio-frequency circuit. Several cases of commercial applications of class B audio amplifiers are mentioned. Several general problems involved in the use of a class B audio amplifier are discussed with the conclusion that such an amplifier will produce more audio power for a given tube complement, at higher efficiency, and with less distortion than amplifiers previously used in commercial applications.*

UNTIL recently, the constant-current system of plate modulation developed by Heising has been the common way by which transmitters were modulated. This system is illustrated by the circuit shown in Fig. 1. The principle of operation in brief is as follows. The total current for both the modulator tube M and the load circuit R, which may be either a self-excited oscillator or an amplifier, flows

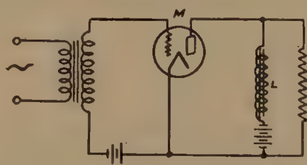

Fig. 1—Class A audio amplifier schematic circuit.

through the modulation choke coil L. This choke coil is one possessing a large amount of self-inductance so that it offers a high impedance to a change in current. Now, as the grid voltage on the modulator tube is varied, the current drawn by the modulator tube changes. As the choke coil offers a high impedance to the change in current, a voltage is built up across the choke coil which will oppose the current change. Suppose that the grid potential is made more negative, the modulator plate current decreases, the modulation choke coil builds up a voltage tending to oppose this change in current; i.e., in a positive direction. This increase in voltage appears at the plate of the modulator tube and across the load R which causes the load circuit to draw more current. The increase in load current is practically equal to the decrease in modulator tube plate current so that the total current, i.e., the cur-

* Decimal classification: R135×R355.8. Original manuscript received by the Institute, November 30, 1932. Presented before Twentieth Anniversary Convention, April 9, 1932, Pittsburgh, Pa.

ent through the modulation choke coil is a constant, hence, the name, onstant-current system. A handy way of plotting the dynamic or nodulation characteristic is to assume changes in plate voltage and urrent in the load circuit. The modulator plate current is assumed to e the difference between the steady current through the modulation hoke coil and the load current. The instantaneous value of grid voltge is then found from the tube characteristics and is the value which vill cause the instantaneous value of plate current to flow at the instananeous value of plate voltage. A sufficient number of such points will llow us to plot the dynamic characteristic. Such a characteristic,

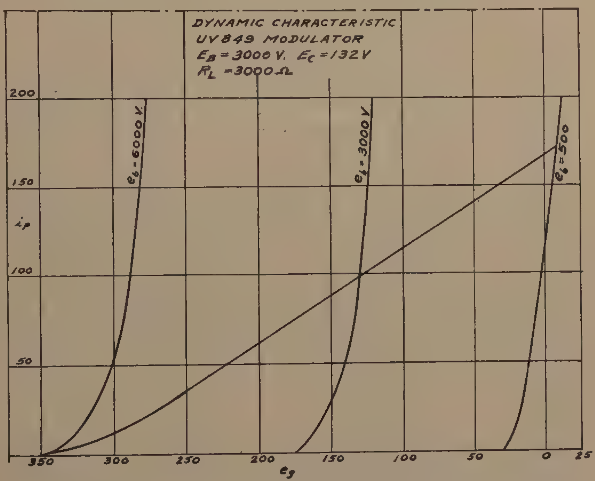

Fig. 2—Class A audio amplifier dynamic characteristic.

shown in Fig. 2, is plotted for a UV-849 as modulator drawing normally 0.1-ampere plate current at 3000 volts, modulating a UV-849 radio tube load circuit which is also drawing 0.1 ampere at 3000 volts. This system of modulation is known as the class A system. Thus, class A operation of a tube can be defined as the operation of a tube so that plate current for the tube never goes to zero at any time during a grid voltage cycle. The limit of operation of such a system occurs at the bend of the dynamic characteristic, as it approaches the condition of zero modulator current, thus entering into the curved portion of the tube static characteristics.

Now, suppose that the fixed bias on the modulator tube grid is increased until zero plate current flows at normal plate voltages. If the grid voltage is increased in a positive direction plate current will flow in the tube. Again the modulation reactor opposes the change in current through the load R, an amount corresponding to the increase in

modulator plate current. If the cycle is continued the grid will eventually have sufficient negative potential to stop all plate current in the tube, and as the potential will increase negatively for the remaining half cycle, the plate current in the tube will not change during the negative half cycle. No change in plate current will result in no change in voltage across the load circuit for the negative half cycle. Now, if

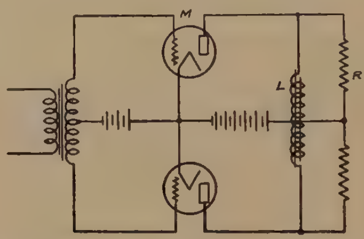

Fig. 3—Class B push-pull audio amplifier schematic circuit.

another tube is connected so that its operation is exactly opposite to the first tube, that is, so that it operates during the half cycle when the first tube is idle, changes in voltage across the load resistance can be made to occur continuously throughout the cycle. Such a circuit is shown in Fig. 3. Here we have both tubes feeding into an inductance which acts like an autotransformer. The voltage developed across the

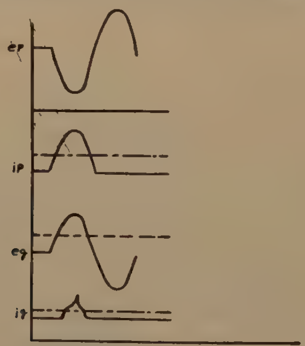

Fig. 4—Relation of plate and grid voltage and current for class B amplifier tube.

impedance of one half the choke coil is auto-induced in the other half, in the same direction. The voltage produced by the operation of one tube is out of phase 180 degrees with the voltage produced by the other so that there is a continuous alternating voltage appearing across the load resistance. This then is the operation of the class B push-pull amplifier. The class B operation of a tube can be defined as the operation so that plate current flows for just one half of a grid-voltage cycle.

As the dynamic characteristic of a tube is practically linear in the region of positive grid potentials, the tube is made to operate with instantaneous positive potentials on the grid, resulting in high values of plate current and, therefore, output load current, and as the tube loss is reduced because of the lower instantaneous values of plate voltage, the efficiency is higher than that of a class A amplifier. Fig. 4 shows the relation of plate voltage, plate current, grid voltage, and grid current for a class B operated amplifier tube. Fig. 5 shows the relation of

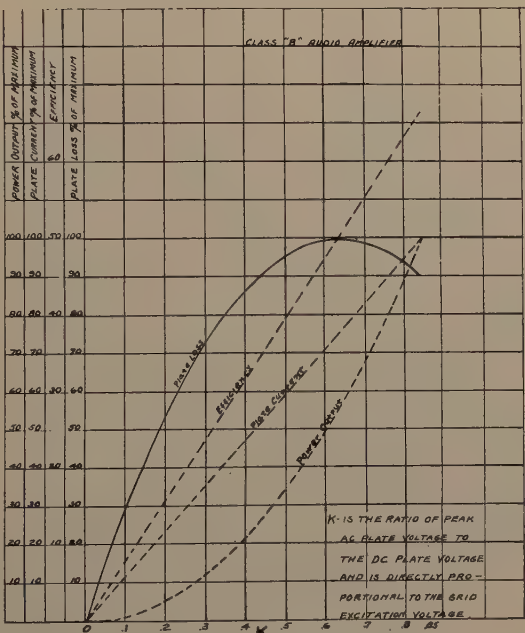

Fig. 5—Relation of output, plate loss, plate current, and efficiency to output voltage.

efficiency, output, plate loss, and plate current for a tube, the ordinates being in per cent and the abscissa being the ratio of the peak alternating component of plate voltage to the steady direct-current plate voltage. Practically, the minimum instantaneous value of plate voltage is limited to 15 per cent of the direct-current plate voltage. Thus, the peak alternating-current component of plate voltage is the remaining 85 per cent. It is interesting to note that the efficiency at this condition is 66.6 per cent. It is also interesting to note that the maximum plate loss does not occur at the maximum output but at a condition of operation which results in 50 per cent efficiency.

The recent rapid development of the so-called class B system of

amplification of audio frequencies has presented many interesting problems in the design and application of this system to high power radio transmitters. The class B system of audio amplification has made available relatively large amounts of audio power at an efficiency which still allows the practical application of this power. Some of the problems and accomplishments accompanying the application will be given here.

The system of class B amplification is not new, having been used in many radio transmitters, but the use of this system for the amplification of audio frequencies is a comparatively recent development. A commercial application appears in a transmitter developed for the U. S. Department of Commerce to be used at various airways stations. This transmitter is rated at 1200 watts for telephone transmission in the radio-frequency band of 190 to 550 kilocycles. The over-all efficiency obtained in the radio amplifier of this unit makes necessary for complete modulation approximately one kilowatt of audio power. This power is obtained by the use of a class B audio amplifier using two UV-849 tubes to deliver the necessary power. Had tubes of this type been used in the old class A system of modulation, ten tubes

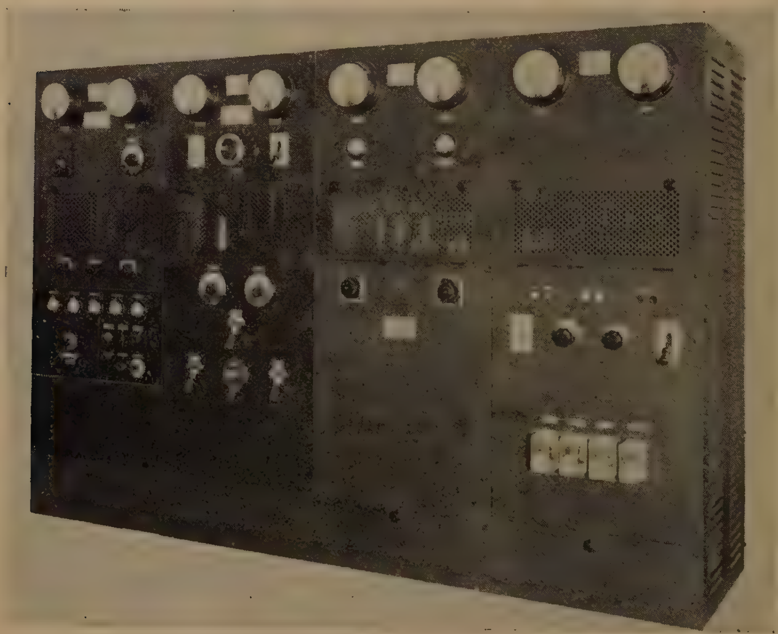

Fig. 6—Department of commerce type TBH radiotelephone and radiotelegraph transmitter.

vould have been necessary to supply the power. In the class A amlifier, there would have been a steady load of 3 kilowatts drawn from he plate supply for the modulator alone. In the new class B sysem, the "no-talk" load is approximately 300 watts, the maximum load s approximately 1800 watts and the average load is of the order of

Fig. 7—Modulator unit for type TBH transmitter

one kilowatt or less with ordinary voice modulation. It is quite obvious therefore that the class B system offers an advantage from the standpoint of power required and also from the standpoint of tube complement. The audio quality of the amplifier under discussion is fully as good as the best broadcast transmitter in existence today. The oscillogram of rectified antenna current showing essentially complete modulation at an audio frequency of 400 cycles will give an idea of the qual-

ity of such a modulator. The total distortion is a result of not only the class B modulator but also five preceding audio-frequency amplifier stages, three of them operating class A and two operating class B. This oscillogram has been analyzed and shows a total audio harmonic distortion of slightly over six per cent. The audio system described is capable of amplifying a signal level of one one-hundredth of a micro-watt up to one kilowatt with a total of only slightly more than six per cent distortion. The total power amplification is 10^{11} times, or one-hundred billion times.

Another example of such a system is now in use at KDKA where an amplifier is in operation which will deliver more than fifty kilowatts of power. This tremendous amount of audio-frequency power is used to plate modulate a radio-frequency amplifier which will deliver fifty

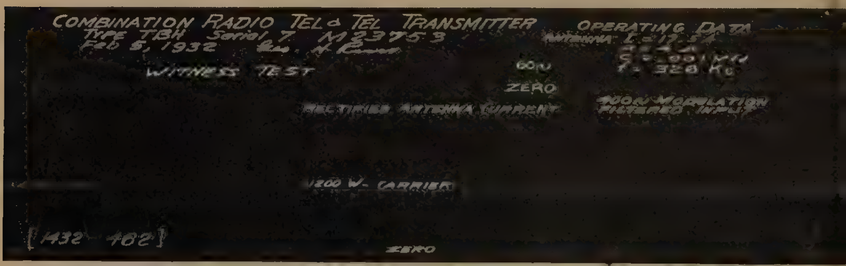

Fig. 8—Oscillogram of rectified antenna current.

kilowatts of carrier power to an antenna. This amplifier is essentially the same as the one previously described with the addition of another stage to bring the power level up to fifty kilowatts. The tube complement of the output audio amplifier is six UV-863 tubes. An equivalent amplifier using UV-848 tubes, similar to the UV-863 except of lower mu and developed for class A operation, would require in a class A circuit at least twenty such tubes to deliver the same amount of power. Such an amplifier would require a steady power of at least 150 kilowatts for the plate supply, whereas the maximum power under steady full load output for the class B system is 75 kilowatts and the average power required during regular program broadcast is of the order of forty to fifty kilowatts. The advantages of the class B system are at once apparent.

The problems associated with the development of the aforementioned equipment have been very complex, involving new lines of thought. A class B amplifier must be thoroughly coupled into the load to which it will deliver its power. This involves the coupling trans-

ormer and the coupling network. As the amplifier is intended to be sed as a modulator, the load is defined as the plate voltage of the nodulated stage, divided by the plate current. For complete modula-on the peak value of the modulating voltage must equal the direct-urrent plate voltage of the modulated stage. Therefore, as the load ircuit is considered to be a pure resistance the peak value of modulat-ng current must equal the direct plate current of the modulated stage.

Fig. 9—Class B modulator unit at KDKA.

Thus, the root-mean-square voltage is equal to 0.707 E_B and the root-mean-square current equals 0.707 I_B. The power delivered by the modulator then is $0.707\, E_B \times 0.707\, I_B = \frac{1}{2} E_B I_B$. For complete modu-ation then the modulator must deliver an amount of power equal to one half the power which it is desired to modulate. For high power work it is deemed best to separate the direct-current and alternating-current components in the load, allowing the direct-current component to flow through a modulation reactor and the alternating-current com-ponent only to flow in the secondary of the modulation transformer.

Fig. 12 shows such a circuit. A little thought will show the reason fo this. The requirements of broadcast equipment today as regards har monic audio distortion are very strict, limiting the arithmetic sum o all harmonics to ten per cent. It is seen then that every precaution mus be taken to assure the linearity of the modulator. If the load direc

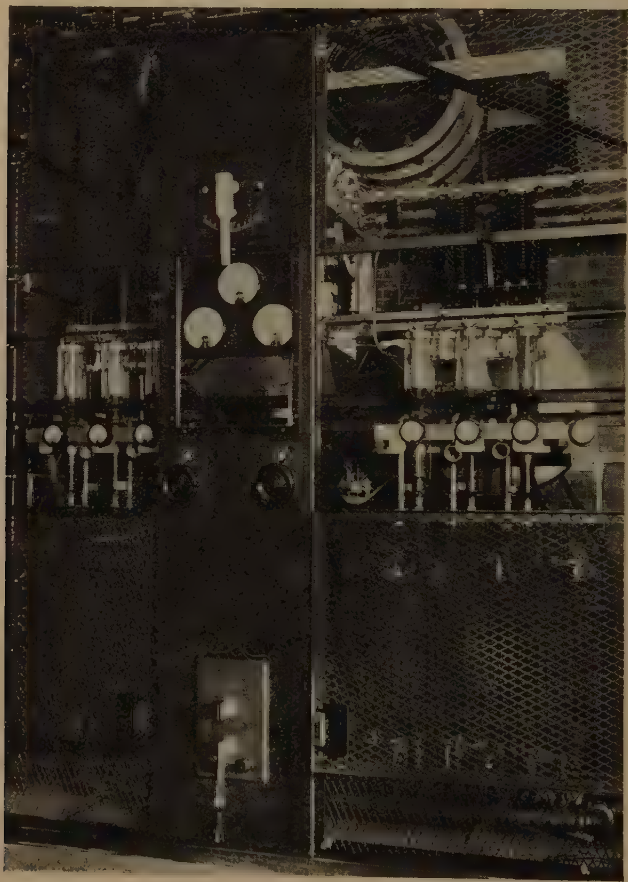

Fig. 10—Modulated amplifier at KDKA.

current were allowed to flow through the secondary of the modulation transformer, it would produce a steady flux in one direction. Now during one half of an audio cycle, the flux caused by the audio voltage would be in the same direction as that caused by the direct current, and for the other half cycle the flux would oppose that caused by the direct current. Thus, the inductance of the primary of the modulation transformer would be reduced for one tube and increased for the other.

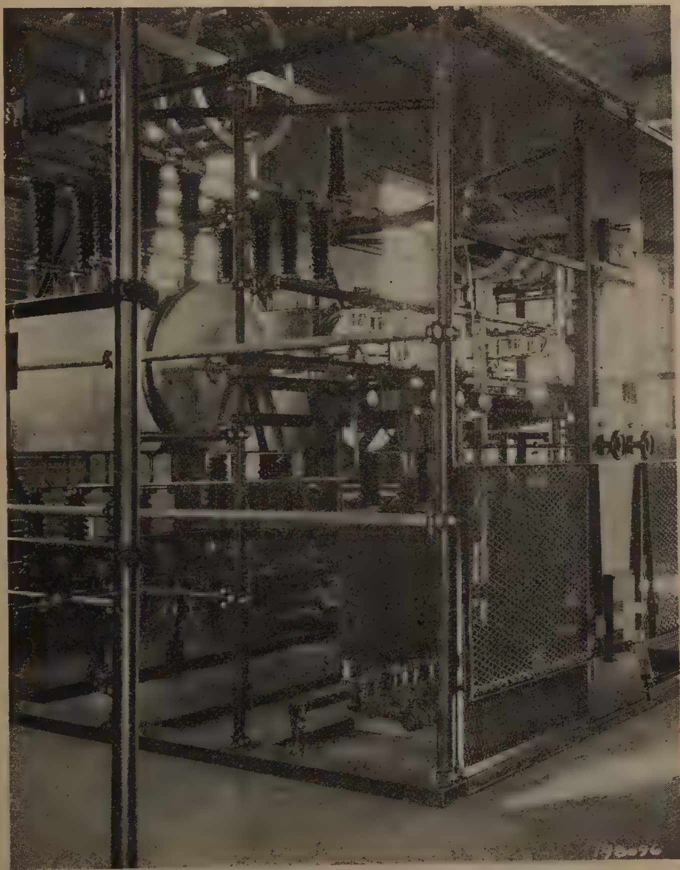

Fig. 11—Modulated amplifier at KDKA.

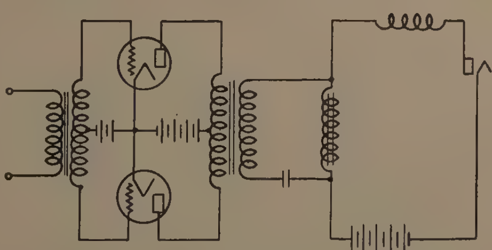

Fig. 12—Schematic modulation circuit using class B push-pull audio amplifier.

While this change in inductance could be made small by having sufficient iron in the core of the transformer, the slight variations might still be sufficient to produce large enough even harmonic components in the output wave to make the use of the equipment impracticable. Therefore, the output circuit was designed to make the transformer carry only the alternating-current component of the load current. From calculations involving tube characteristics it can be shown that

Fig. 13—Modulation transformer at KDKA.

the maximum practical primary voltage is 1.2 times the direct-current plate voltage of the modulators. That is, the root-mean-square alternating-current voltage across the full primary winding is equal to 1.2 times the modulator direct-current plate voltage. Knowing the load voltage which it is desired to modulate, the root-mean-square secondary voltage is 0.707 times the direct-current load voltage for 100 per cent modulation. Thus, we know both the primary and secondary voltages at full load and also the load current.

Thus, we have the general size requirements of the transformer determined.

The transformer used to supply the audio power to the plate circuit of the radio-frequency amplifier at KDKA is certainly unique, and as the problems and their methods of solution applicable to this transformer are typical of the modulation transformer for a transmitter of smaller power, a description of it is given here.

The audio transformer operates over a wide range of frequencies. In order that its response will be uniform over a wide range of frequencies it is necessary that it have low reactance between the input and output windings and that its internal capacitance be low. Also, since the input may contain a direct component of current provisions must be made to prevent the core of the unit from becoming saturated and causing serious magnetizing current distortions. To obtain low reactance the transformer was made very large in iron section and low in number of turns, the number of turns being fixed by the maximum permissible reactance at the highest frequencies and the cross section of the core being fixed by the minimum operating frequencies. Low internal capacitance was obtained by making the windings of a large number of pancake coils connected in series, thus approximating a banked winding. The various sections of the windings were then so connected as to give balanced capacitance of the two ends to ground. In order to prevent direct-current saturation of the core an air gap was placed in the core of such length that the maximum direct-current flux density would approximately equal the maximum alternating-current flux density. This gives a good compromise between total magnetizing current and distortion in magnetization.

The external circuits of a class B amplifier present several interesting problems. For instance, the power supply source must have good regulation. The plate current of a class B amplifier is not steady but varies directly as the input voltage. Suppose at a certain audio input voltage the plate supply voltage is a definite value. Now for proper reproduction of broadcast programs it is necessary that the percentage of modulation follow directly the changes in audio voltage input. If the plate supply voltage drops, the output voltage will not double but will be some value less than twice the original value. If the regulation is bad then it is apparent that the variations in output voltage or per cent modulation will not follow directly the input voltage, and the resulting output will not be a true reproduction of the input. This problem is not as serious as might at first be thought, however, because these changes do not take place during an audio cycle providing there is sufficient capacity in the plate circuit to carry over peaks of plate

current at an audio-frequency rate. The changes in level occur with phrase modulation, that is, changes of program level from a low percentage of modulation to a high percentage of modulation. As a ten per cent reduction in voltage represents a change of less than one decibel, it is seen that a reasonable amount of regulation in the power circuits can be allowed. One point which makes possible the use of ordinary plate supply equipment is the fact that usually the supply carries not only the modulator but also several other parts of the circuit, which makes the modulator load only a portion of the total load. Thus, the variations of load on the supply source are not relatively large.

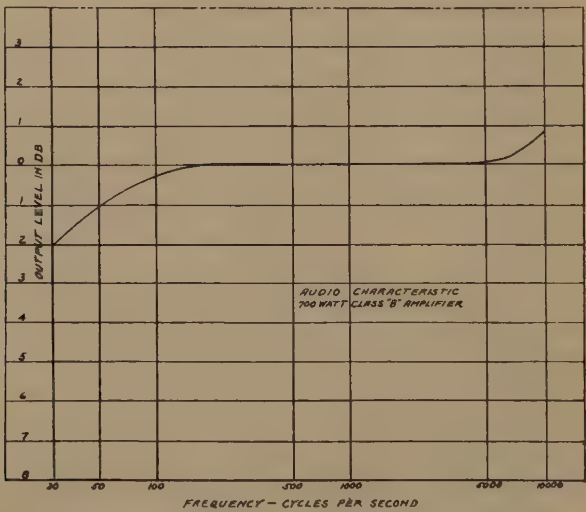

Fig. 14—Typical audio characteristic class B push-pull audio amplifier.

Another point in the external circuit which must be carefully considered is the supply of audio voltage to excite the grids of the modulators. The grid circuit absorbs quite an appreciable amount of power during peaks of modulation, while at somewhat reduced levels the grids may take zero power. The driving stage must not be seriously affected by the change of load due to the effective resistance of the modulator grids varying from infinity to a low value, depending upon the type of tube and the conditions of operation. In addition the bias voltage circuit must be designed so that the variations in bias voltage due to the flow of grid current will not be appreciable. Both of the above-mentioned items, that is, the ability of the audio supply source to supply peak grid power, and the variation in bias voltage, affect the output of the modulator during an audio-frequency cycle, and therefore result in distortion.

The audio characteristic of a class B audio modulating system can be made as good as that of an old type constant-current system. Fig. 14 shown here is an audio characteristic taken on a class B system showing the variation in output level plotted in decibels as ordinates as a function of the frequency plotted as abscissas. This curve was taken with the audio input voltage maintained at a constant level.

The quantitative values which can be assigned to the various items which govern the quality of a modulation system are a function of the quality required. It is safe to say that a properly designed class B audio amplifier will supply more audio power with less distortion (for the same tube complement) than any type of audio amplifier previously in commercial use.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

THE INTERDEPENDENCE OF FREQUENCY VARIATION AND HARMONIC CONTENT, AND THE PROBLEM OF CONSTANT-FREQUENCY OSCILLATORS*

By

Janusz Groszkowski
(Radio Institute, Warsaw, Poland)

Summary—*The investigations of the frequency variations in oscillating systems with negative nonlinear resistance were based until now on the fundamental differential equation of the simple oscillatory circuit.*

This method, although very exact, is not always sufficiently simple and even not always possible, especially if it concerns more complicated schemes. The symbolic calculus, although very simple in use, if applied to the nonlinear systems, gives only approximate results. The following paper represents the operation of nonlinear systems from a different point of view. Here the symbolic calculus has been used throughout in a complete and exact manner, by employing it for the fundamental frequency as well as for all harmonic frequencies which appear in the system. This could be done owing to the investigation of the negative resistance operation from the energy point of view.

It was taken into consideration, that in the negative resistance characteristics $i = f(v)$, i must be the univocal function of v, and therefore the area described by the instantaneous point of work during one cycle of the fundamental oscillation must be zero or,

$$\oint idv = 0.$$

On the other hand i and v can be considered, with regard to the external circuit connected to the negative resistance, as the sum of harmonic currents and voltages. In this way we obtain the formulas which allow the interdependence of the frequency variation and the content of harmonics to be determined. It appears that these harmonics, whose amount varies with the change of operative conditions (supply voltages etc.) of the system, are just responsible for the frequency variation.

As a consequence, some methods of frequency stabilization can be deduced. Many theoretical investigations have been experimentally verified.

Introduction

THE problem of the generation of electrical oscillations by means of systems with negative nonlinear resistance has been the subject of only a few investigations.[1,2,3]

In these mathematical treatments the following differential equation was taken as a starting point:

* Decimal classification: R140. Original manuscript received by the Institute, December 22, 1932. Published in Polish in *Wiadomości i prace Instytutu Radjotechnicznego*, vol. 4, nos. 5 and 6, (1932).
[1,2,3] Refer to Bibliography.

$$\frac{d^2v}{dt^2} + f'(v)\frac{dv}{dt} + \omega^2 v = 0$$

eferred to the nonlinear system, given by

$$f'(v) = -\alpha + \beta v + \gamma v^2 + \delta v^3 + \cdots .$$

Hence the oscillation equation

$$v = \phi(t)$$

as well as the angular frequency ω for given operative conditions are determined.

This method, however, is very laborious, especially with regard to more complicated circuits, and the results obtained cannot be represented by means of sufficiently simple expressions and, therefore, do not always render it possible to draw conclusions for practical purposes.

The present work has in view the representation of the nonlinear system operation in a somewhat different way by help of the symbolic electrical calculus. The method given below throws some light on the mechanism of the frequency variations which occur in the generating systems, and allows one to obtain in a simple way the formulas for any circuit.

It can be proved that the frequency variations are related strictly to the alteration of the content of harmonics which appear when the operative conditions of the oscillation system are changed; these harmonics being justly responsible for the frequency variation.

Thus, the evident consequences concerning the independence of the oscillation frequency from the variation of the operative conditions can be deduced.

This conception, based on the rôle of harmonics, also allows the different methods of nonmechanical stabilization of the generators to be discussed from the same point of view.

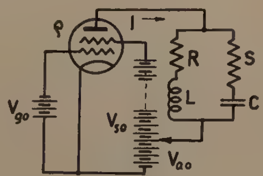

Fig. 1

A. Operation in Ideal Conditions on the Linear and Infinite Characteristics

The operation of an oscillator which consists of a circuit L, C, R, S connected with a negative resistance ρ (as, for example, a dynatron—

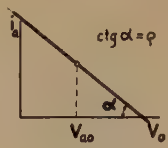

Fig. 2

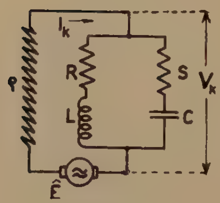

Fig. 3

Fig. 1), this negative resistance being given by a falling linear and infinite characteristics, (Fig. 2) is determined—in the stationary state—by the symbolic equation (Fig. 3):

$$\mathrm{I}\left[\rho + \frac{(R + j\omega L)\left(S - j\dfrac{1}{\omega C}\right)}{R + S + j\omega L - j\dfrac{1}{\omega C}}\right] = \mathrm{I}(A + jB) = \mathrm{E} \to 0 \qquad (1)$$

From (1) results two equations

$$A = 0 \qquad (2)$$

$$B = 0 \qquad (3)$$

which are independent of the current amplitude. Equation (3) determines the frequency of the oscillations

$$\omega^2 = \frac{1}{LC}\,\frac{1 - \dfrac{R^2C}{L}}{1 - \dfrac{S^2C}{L}}. \qquad (4)$$

This formula can be written

$$\omega^2 = \omega_0^2 \frac{1 - \delta_r^2}{1 - \delta_s^2} \qquad (5)$$

if we denote

$$\omega_0^2 = 1/LC \qquad (6)$$

$$\delta_r = R/\omega L \cong R/\omega_0 L \qquad (7)$$

$$\delta_s = S\omega C \cong S\omega_0 C. \qquad (8)$$

In the case where $\delta_r \ll 1$ and $\delta_s \ll 1$, the frequency becomes

$$\omega^2 \cong \omega_0^2 = \frac{1}{LC}. \tag{9}$$

From (2) we obtain the condition

$$\rho = -\frac{L/C + RS}{R + S} \cong -\frac{L}{C(R + S)} \tag{10}$$

which must be fulfilled in the stationary state of work of the oscillator. This condition (10) shows that for a given circuit there exists only one value of ρ to which corresponds the stationary state of work on the linear and infinite characteristics. Any other stable state is impossible and cannot be examined by help of these formulas.

Particular Cases

If $S = 0$, the formulas (10) and (4) become

$$\rho = -\frac{L}{RC} \tag{11}$$

$$\omega^2 = \omega_0^2\left(1 - \frac{R^2C}{L}\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \omega_0^2\left(1 + \frac{R}{\rho}\right). \tag{12}$$

The oscillation frequency is slightly below the natural frequency of the circuit; it diminishes with the increase of R or with the decrease of ρ.

If $R = 0$, (10) and (4) become[4]

$$\rho = -\frac{L}{CS} \tag{13}$$

$$\omega^2 = \frac{\omega_0^2}{1 - \frac{S^2C}{L}} = \frac{1}{\left(1 + \frac{S}{\rho}\right)^2}\left[\frac{1}{LC} - \frac{S^2}{L^2}\right] = \frac{\omega_0^2}{1 + \frac{S}{\rho}}. \tag{14}$$

The oscillation frequency is slightly above the natural frequency of the circuit; it grows with the increase of S or with the decrease of ρ.

If $R = S = T$, (10) and (4) are

$$\rho = -\frac{L}{2TC} \tag{15}$$

$$\omega^2 = \omega_0^2 = \frac{1}{LC}. \tag{16}$$

The frequency does not depend upon resistances; it is determined solely by L and C of the circuit.

The operation of the oscillator on the linear and infinite characteristics is distinguished by the purely sinusoidal oscillations; i.e., the currents and voltages do not possess any harmonics.

The Experimental Results.

In order to prove (4), an oscillatory system consisting of a circuit and a screen valve (Tungsram S 407)—which worked as pliodynatron—was examined. The system was always adjusted to the critical condition of work (on the limit of generation) by means of the inner grid bias. The oscillation circuit consisted of the self-inductance

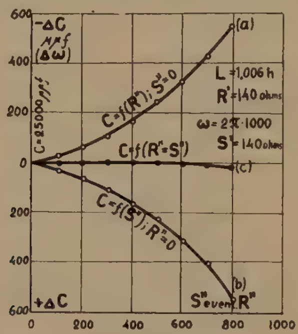

Fig. 4

$L = 1.006$ henrys, $R' = 140$ ohms and the capacity, $C = 25{,}000$ $\mu\mu$f. This capacity was adjusted on zero beat with a 1000-cycle constant-frequency source, in order to maintain the generated frequency constant. The variation of the capacity permitted one to determine the corresponding frequency variation.

It is possible to prove (16) by adding the resistance $S' = 140$ ohms in the capacity arm; the experiment, indeed, gives in this case exactly $C = 25{,}000$ $\mu\mu$f. When the resistance R'' or S'' were added in the L-arm or C-arm or simultaneously $R'' = S''$ into both of them, there were obtained curves a, b, and c represented on Fig. 4.

These results agree well with the theoretical calculations, with the aid of (4).

B. Operation in Real Conditions on the Curved and Limited Characteristics

In reality, the operation on the linear and infinite characteristics is not possible; one can only more or less approach these conditions,

as the characteristics are neither linear nor infinite. In the most favorable case, as, for example, in the dynatron, we have at our disposal a curve with an inflection, in the middle of which we can choose the initial point of work (Fig. 5). The resistance ρ here is not constant, it in-

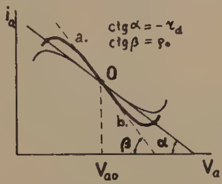

Fig. 5

creases with the increase of amplitude from a minimum value $\rho = \rho_0$. If the oscillation ceases to be sinusoidal, the symbolic calculus cannot be employed in the simple manner for the solution of the problem.

As to the phenomena on the fundamental frequency, the conception of a dynamic resistance being a function of the amplitude can be suggested:

$$\rho_d = f(I) \tag{18}$$

which becomes

$$\rho_d \doteq \rho_0 \tag{19}$$

only for $I \to 0$.

At the same time (2) and (3) which were established for the fundamental frequency, and which result from the principal symbolic equation, are not exact any more, for they do not take into account the harmonics. In order to take account of the direction of the variations which occur under these conditions, we can write (2) and (3) in a general form:

$$A_{(I)} = 0 \tag{20}$$

$$B_{(I)} = 0 \tag{21}$$

remembering that $A_{(I)}$ and $B_{(I)}$ are functions of the amplitude, because otherwise we would not obtain the stationary operational state of the dynatron.

Now let us suppose that the dynatron oscillator is adjusted for the critical condition:

$$r_d = \frac{L}{(R+S)C}; \tag{22}$$

i.e., the straight line (Fig. 5) representing the dynamic resistance of the circuit is tangential to the characteristics of the dynatron. If now,

by changing the slope of the characteristics (e.g., by means of the bias V_g) we bring it† to the intersection with the straight line, we shall obtain a stationary state which is given by the relation

$$\rho_d = - r_d$$

ρ_d is here the dynamic resistance of the dynatron, the minimum resistance of which is ρ_0.

It is obvious that the variation of ρ_d will produce a frequency variation, this frequency being a function of the amplitude.

The Experimental Results.

The curves of Fig. 6 show—as an example—the frequency variations (expressed in terms of capacity variations which are necessary for their compensation) as a function of the minimum dynatron resistance, this resistance being changed by means of the bias V_g. It can be seen that the frequency decreases in order to obtain the same frequency, the capacity of the circuit must be decreased when the system is departing from the critical state. This frequency variation is practically independent of the resistance distribution in both arms of the circuit.

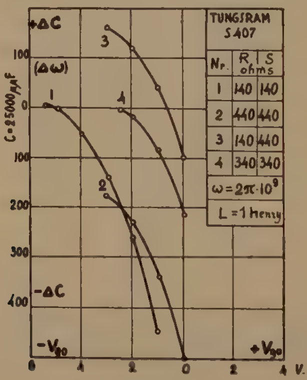

Fig. 6

The Frequency of the System.

In case the characteristics of the negative resistance are not linear, (i.e. the phenomena are not purely sinusoidal but contain harmonics) (1), based only on the fundamental frequency, does not represent sufficiently the exchanges of the energy of the system: but only these ex-

† We assume that the characteristic curve is relatively regular; i.e. it can intersect the straight line in three points only.

changes can be taken as a basis in considering the systems in which the free oscillations of the electric charge occur.

The investigation given below and based actually upon the energy equations, allows the interdependence, which exists between the frequency variation and the operative conditions of the dynatron oscillator, to be determined. It appears, as was already mentioned above, that the frequency variation is closely related to the amount of harmonics.

In the stationary state of operation, if the negative resistance is purely of an electronic nature (as, for example, a dynatron) it can supply the oscillatory circuit with the energy which has to compensate the losses occurring in it (the "real" energy). The "imaginary" energy, however, which corresponds to the oscillation of the electric charge between the capacity C and the inductance L of this circuit must be balanced within the circuit itself, because the negative resistance, which has neither capacitive nor inductive properties is not able to accumulate the energy.

Thus, the characteristics of such a negative resistance

$$i = f(v) \tag{23}$$

must be a curve for which the current i is the univocal function of the voltage v. Then the area described by the instantaneous point of work in the (i, v) coördinate system, must be zero for a total period of the fundamental frequency.

This condition is given by

$$\oint idv = 0. \tag{24}$$

As the negative resistance is connected directly to the oscillatory circuit, there must be a relation, given by (24), between the instantaneous values of voltage and current in this circuit. This current and this voltage, being related to an oscillatory system in which any external alternating electromotive force does not act, must form, in the stationary state, the harmonic series of frequency. Thus, they have the following form:

$$v = \sum_{k=1}^{k=\infty} v_k = \sum_1^\infty V_k \sin(k\omega t + \alpha_k) = \sum_1^\infty \mathrm{V_k} \tag{25}$$

$$i = \sum_{k=1}^{k=\infty} i_k = \sum_1^\infty I_k \sin(k\omega t + \beta_k) = \sum_1^\infty \mathrm{I_k} \tag{26}$$

V_k and I_k are amplitudes, $\mathrm{V_k}$ and $\mathrm{I_k}$ = symbols of voltages and currents, α_k and β_k = their phase angles, ω = fundamental angular frequency, k = order of harmonics.

On the other hand, between V_k and I_k there must exist a relation, determined by the impedance of the circuit Z_k for the harmonics "k," and namely, in the symbolic form:

$$\mathrm{I_k} = \frac{\mathrm{V_k}}{Z_k}. \tag{27}$$

Equations (24) and (27) allow the relation between frequency variation and the content of harmonics to be determined. For this purpose let us transform (24) into the form applied for the symbolic calculus. From (25) we get

$$dv = \sum_1^\infty k\omega V_k \cos (k\omega t + \alpha_k) dt \tag{28}$$

and, therefore, (24) becomes

$$\oint idv = \int \left[\sum_1^\infty I_k \sin (k\omega t + \beta_k) \right] \left[\sum_1^\infty k\omega V_k \cos (k\omega t + \alpha_k) \right] dt. \tag{29}$$

Integrating (29) for one total period of the fundamental frequency ($k=1$); i.e., in the limits from 0 to 2π, we obtain the expression (see Appendix I)

$$\sum_1^\infty \pi k I_k V_k \sin (\beta_k - \alpha_k) = 0 \tag{30}$$

or,

$$\sum_1^\infty k V_k I_k \sin \phi_k = 0 \tag{31}$$

if we denote the phase angle between voltage and current of the harmonics

$$\phi_k = \beta_k - \alpha_k. \tag{32}$$

According to the definition, generally used in electrical engineering, the expression

$$\tfrac{1}{2} V_k I_k \sin \phi_k \tag{33}$$

is the imaginary power of the alternating current.

This power can be represented symbolically as the imaginary component of the expression (see Appendix II)

$$\tfrac{1}{2} [\mathrm{V_k I_k}]_{im}. \tag{34}$$

Symbolically (31) becomes:

$$\sum_{1}^{\infty} k \mid \mathrm{V_k I_k} \mid_{im} = 0. \tag{35}$$

Using (27), we represent (35) in the form

$$\sum_{1}^{\infty} V_k^2 \left|\frac{k}{Z_k}\right|_{im} = 0. \tag{36}$$

After eliminating the fundamental we have:

$$V_1^2 \left|\frac{1}{Z_1}\right|_{im} + \sum_{2}^{\infty} V_k^2 \left|\frac{k}{Z_k}\right|_{im} = 0. \tag{37}$$

If we put the ratio of the harmonic voltage to the fundamental

$$m_k = \frac{V_k}{V_1} \tag{38}$$

equation (37) can be rewritten as follows:

$$\left|\frac{1}{Z_1}\right|_{im} + \sum_{2}^{\infty} \left|\frac{k}{Z_k}\right|_{im} \cdot m_k^2 = 0. \tag{39}$$

Analogically, introducing the current harmonics

$$n_k = \frac{I_k}{I_1} \tag{40}$$

we obtain (35) in the form

$$\mid Z_1 \mid_{im} + \sum_{2}^{\infty} \mid kZ_k \mid_{im} \cdot n_k^2 = 0. \tag{41}$$

Let us apply (39) for the simplest resonant circuit, shown in Fig. 3, for which the ratios m_k are known. Its impedance for the frequency $k\omega$ is

$$Z_k = \frac{(R + jk\omega L)\left(S - j\dfrac{1}{k\omega C}\right)}{R + S + jk\omega L - j\dfrac{1}{k\omega C}}. \tag{42}$$

Introducing (8), (9), and (10) we transform (42)

$$\frac{1}{Z_k} = \frac{1}{\omega L} \frac{\left(\frac{\omega}{\omega_0}\right)^2 (\delta_r + jk) + \left(\delta_s - j\frac{1}{k}\right)}{(\delta_r + jk)\left(\delta_s - j\frac{1}{k}\right)}. \tag{43}$$

Hence, we eliminate

$$\left|\frac{k}{Z_k}\right|_{im} = \frac{1}{\omega L} \frac{\left(\frac{\omega}{\omega_0}\right)^2 (k^2 + \delta_r^2) - (k^2\delta_s^2 + 1)}{(\delta_r^2 + k^2)(k^2\delta_s^2 + 1)} k^2. \tag{44}$$

If,

$$\delta_r \ll 1, \ \delta_s \ll 1, \text{ and } k^2\delta_s^2 \ll 1,$$

equation (44) can be written

$$\left|\frac{k}{Z_k}\right|_{im} \cong \frac{1}{\omega L}\left[\left(\frac{\omega}{\omega_0}\right)^2 \cdot k^2 - 1\right]. \tag{45}$$

For the fundamental ($k=1$)

$$\left|\frac{1}{Z_1}\right|_{im} = \frac{1}{\omega L}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right] \tag{46}$$

whilst for the harmonics (as $\omega \cong \omega_0$ and $k \neq 1$) we can admit

$$\left|\frac{k}{Z_k}\right|_{im} \cong \frac{1}{\omega L}(k^2 - 1) \tag{47}$$

and, therefore, (39) will be

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 \cong - \sum_2^\infty (k^2 - 1) m_k^2. \tag{48}$$

The left side of this equation can be represented as follows:

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 \cong 2\frac{\Delta\omega}{\omega} \tag{49}$$

if we denote by $\Delta\omega$, the frequency variation caused by the presence of harmonics,

$$\Delta\omega = \omega - \omega_0. \tag{50}$$

Equation (48) becomes

$$\frac{\Delta\omega}{\omega_0} = -\frac{1}{2} \sum_{k=2}^{k=\infty} (k^2 - 1) m_k^2. \tag{51}$$

After expanding the sum in (51), we obtain the formula, which is in full agreement with the results obtained by Appleton and Greaves[3] (see Appendix III)

$$\frac{\Delta\omega}{\omega_0} = -\tfrac{1}{2}(3m_2^2 + 8m_3^2 + 15m_4^2 + \cdots). \tag{52}$$

If the content of higher harmonics is small (m_4, m_5, etc. $\ll 1$), we can replace (52) by an approximative formula

$$\frac{\Delta\omega}{\omega_0} \cong -a\sum m^2 \cong -a\cdot\sigma^2 \tag{53}$$

where,

$$\sigma = \sqrt{\sum m^2} = \sqrt{\frac{V_2^2 + V_3^2 + \cdots}{V_1^2}} \tag{54}$$

is the coefficient of content of harmonics.

Physically, the influence of the content of harmonics on the frequency variation can be explained as follows:[5] If the oscillations are purely sinusoidal, the fundamental frequency (neglecting the resistances) is determined by (9), and, therefore, the energy distribution in both arms is equal. When the harmonics appear, the currents corresponding to them flow chiefly through the arm C, and therefore they increase the electrostatic energy of this arm in comparison with the arm L. In order to keep the energy equal in both arms, the fundamental frequency must slightly diminish itself in respect to the frequency given by (9). The current in the arm L then increases slightly, in order to allow its electromagnetic energy also to increase suitably.

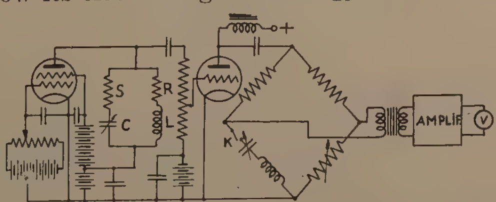

Fig. 7

The Experimental Results.

The arrangement shown in the Fig. 7, served for the purpose of the experimental proof of (51). The well-known bridge method[6] was employed for the measurement of the content of harmonics. The frequency variation was expressed in terms of the capacity variation. The change of the operative conditions was obtained by varying the bias of the inner grid of the dynatron.

The results of measurements for several different circuits and anode voltages are represented in Fig. 8. It is seen that the points are placed along a straight line to which corresponds the coefficient $a \leqq 0.8$. The agreement between the theory and the experiment may be judged as

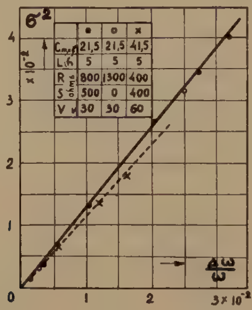

Fig. 8

fully satisfactory, if the difficulties which occur in this kind of measurement are duly appreciated.

C. Generalization of the Problem

The above consideration can be generalized on the triode oscillator with back coupling. We take for the basis of our reasoning the fact that the emission current ($i_e = i_a + i_g$) in the triode is the univocal function of the equivalent potential $v_e = \mu u_g + v_a$ of the electrodes; i.e. that the characteristics

$$i_e = i_a + i_g = f(v_e) = f(\mu u_g + v_a) \tag{55}$$

should be an "arealess" curve.

Hence it must be here

$$\oint i_e dv_e = 0. \tag{56}$$

Equation (56) can be written symbolically

$$\sum_1^\infty k \left| (\mu U_k + V_k) I_k' \right|_{im} = 0 \tag{57}$$

where V_k = the anode voltage, U_k = the grid voltage, I_k' = the emission current, and μ = the amplification coefficient (Fig. 9). Evidently,

$$I_k' = I_k + I_k'' \tag{58}$$

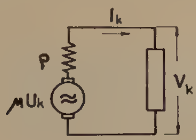

Fig. 9

if I_k denotes the anode current and I_k'' denotes the grid current. For the systems without grid current $I_k''=0$, $I_k'=I_k$ and, therefore, (57) becomes

$$\sum_1^\infty k \left| (\mu U_k + V_k) I_k \right|_{im} = 0. \tag{59}$$

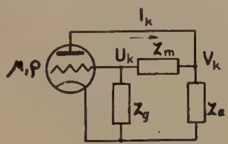

Fig. 10

For the general scheme shown in Fig. 10, we have

$$U_k = -V_k \frac{Z_g}{Z_g + Z_m}; \quad I_k = V_k \frac{Z_a + Z_m + Z_g}{Z_a(Z_m + Z_g)} \tag{60}$$

hence (59) becomes

$$\sum_1^\infty k \left| \frac{[Z_m - (\mu - 1)Z_g][Z_a + Z_g + Z_m]}{(Z_g + Z_m)^2 Z_a} \right|_{im} \cdot m_k^2 = 0. \tag{61}$$

If the resistances in the impedances Z can be neglected; i.e. if

$$Z = R + jX \cong jX, \tag{62}$$

equation (61) can be written

$$\sum_1^\infty k \frac{[X_m - (\mu - 1)X_g][X_a + X_g + X_m]}{(X_g + X_m)^2 X_a} m_k^2 = 0. \tag{63}$$

Resolving this equation for some circuit in use such as Meissner, Hart-

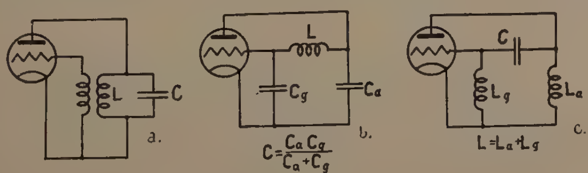

Fig. 11

ley or Colpitts (Fig. 11), we obtain the following approximate formulas:

Meissner:

$$\frac{\Delta\omega}{\omega_0} = -\frac{1}{2}\sum_2^\infty (k^2 - 1)m_k^2. \tag{64}$$

Hartley:

$$\frac{\Delta\omega}{\omega_0} = -\frac{1}{2}\frac{\left(\frac{L_g}{L} - 1\right)^2}{\left[(\mu - 1)\frac{L_g}{L} + 1\right]} \cdot \sum_2^\infty \frac{\left[(\mu - 1)k^2\frac{L_g}{L} + 1\right]}{\left(k^2\frac{L_g}{L} - 1\right)^2}(k^2 - 1)m_k^2 \tag{65}$$

Colpitts:

$$\frac{\Delta\omega}{\omega_0} = -\frac{1}{2}\frac{\left(\frac{C_g}{C} - 1\right)^2}{\left[(\mu - 1) + \frac{C_g}{C}\right]} \sum_2^\infty \frac{\left[(\mu - 1) + k^2\frac{C_g}{C}\right]k^2}{\left(k^2\frac{C_g}{C} - 1\right)^2}(k^2 - 1)m_k^2. \tag{66}$$

For $\mu=0$, $L_g=0$, and $C_s=\infty$ formulas (65) and (66) are transformed into (51) or (64). The same has already been obtained for the dynatron and for the Meissner arrangement.

This result agrees entirely with the expression for the amplitude of the third harmonics obtained by Rocard,[7] with the help of the differential equations.

D. Constant Frequency Generators

In consequence of the above investigations, new possibilities result in the construction of constant-frequency generators; also the operation of many existing constant-frequency systems can be explained. It is obvious that the frequency of the system remains constant, notwithstanding the variation of the operative conditions (for instance, the supply voltages) if there be no harmonics at all, or if their amount

does not change. In other words, that will take place only in the case when the imaginary energy distribution in the oscillatory circuit does not vary with the variation of the operative conditions.

Thus, the ways of proceeding in the establishment of the constant-frequency generators are as follows:

(1) the operation without harmonics; i.e. close to the critical state,

(2) the removing of the harmonics from the oscillatory circuit,

(3) the equalizing of the imaginary energy distribution of harmonics.

(1) *The operation without harmonics.* In order to obtain and to maintain the critical state, the operative conditions must be kept constant, the circumstance which is equivalent already to the maintenance of the constant frequency. This way, therefore, is not of any

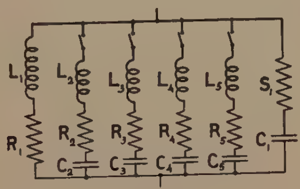

Fig. 12

special interest. More interesting, however, are the arrangements which allow maintenance of the critical state of operation in an automatic way.

(2) *The removal of harmonics.* The simplest way to remove harmonics consists of the application of a number of series filters (Fig. 12), tuned to the corresponding harmonics. If these circuits are sufficiently good, then, owing to their shunt-trapping action, the harmonic voltages on the oscillatory circuit terminals will be considerably reduced, $|m_k|_{k=2}^{k=\infty}=0$, and therefore we receive from (53) $\Delta\omega/\omega \cong 0$.

In practice it is sufficient to apply a few filters for the lowest harmonics only ($k=2$, 3, 4, 5); the higher ones are shunted sufficiently by the capacity of the oscillatory circuit. Now it is evident why the frequency of the generating system, whose circuits have a large capacity, are less subjected to the variation of the operative conditions.

(3) *The equalizing of the energy distribution.* In the simple oscillatory circuit consisting of the inductive arm and the capacitive arm (Fig. 3), the appearing harmonics cause the preponderance of the electrostatic energy in the arm C. For that reason the fundamental frequency must diminish itself slightly in order to produce the proper balance.

The frequency will, therefore, always decrease here with the appearance of harmonics.

Enclosing in the arm C a small inductance N (Fig. 13) we can equipoise the excess of the electrostatic energy of this arm by the electromagnetic energy in the inductance N. It is evident that the arm NC always presents capacitive impedance for the fundamental frequency,

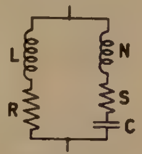

Fig. 13

but for the harmonics it can present a capacitive or inductive impedance according to the order of the harmonics and to the value of the product NC.

Taking the fundamental equation (39) as a basis, we can deduce the relation

$$\frac{\Delta\omega}{\omega} = f(m_k^2). \tag{67}$$

With this in view we denote

$$C(L+N) = \frac{1}{\omega_1^2}; \quad CN = \frac{1}{p^2\omega_1^2}. \tag{68}$$

(It is obvious that $p \geqq 1$, because $p^2 = (L+N)/N$.)

We have for such a circuit

$$\left|\frac{k}{Z_k}\right|_{im} = \frac{1}{\omega L} \frac{k^2\left(\dfrac{\omega}{\omega_1}\right)^2 - 1}{\dfrac{k^2}{p^2}\left(\dfrac{\omega}{\omega_1}\right)^2 - 1} \tag{69}$$

and therefore (39) becomes

$$\frac{\left(\dfrac{\omega}{\omega_1}\right)^2 - 1}{\dfrac{1}{p^2}\left(\dfrac{\omega}{\omega_1}\right)^2 - 1} = -\sum_2^\infty \frac{k^2\left(\dfrac{\omega}{\omega_1}\right)^2 - 1}{\dfrac{k^2}{p^2}\left(\dfrac{\omega}{\omega_1}\right)^2 - 1} \cdot m_k^2. \tag{70}$$

Because $\omega \cong \omega_1$, equation (70) transforms itself into

$$\left(\frac{\omega}{\omega_1}\right)^2 - 1 = -\left(\frac{1}{p^2} - 1\right) \sum_2^\infty \frac{k^2 - 1}{\left(\frac{k}{p}\right)^2 - 1} \cdot m_k^2. \tag{71}$$

Taking into consideration that

$$\left(\frac{\omega}{\omega_1}\right)^2 - 1 \cong 2\frac{\Delta\omega}{\omega_1} \tag{72}$$

we obtain, after transformation,

$$\frac{\Delta\omega}{\omega_1} = -\frac{1}{2} \sum_2^\infty \frac{p^2 - 1}{p^2 - k^2}(k^2 - 1) \cdot m_k^2. \tag{73}$$

In the case where $p = \infty$ or $N = 0$, i.e., for a simple circuit (Fig. 3) the expression (73) changes into the known formula (64).

The examination of (73) shows that the coefficients are negative or positive according to the harmonic order "k."

Thus, for example, (if $p > 1$) for $k <$ p we have

$$\frac{p^2 - 1}{p^2 - k^2} > 0$$

while, for $k > p$

$$\frac{p^2 - 1}{p^2 - k^2} < 0.$$

For $1 < p < 2$ (i.e., when the arm NC presents for all harmonics an inductive impedance), we obtain, supposing $p \cong 1.7$, $(p^2 = 3)$, the expression

$$\frac{\Delta\omega}{\omega} = 3m_2^2 + \frac{4}{3}m_3^2 + \frac{15}{13}m_4^2 + \cdots \tag{74}$$

which shows that the frequency increases with the increase of the content of harmonics.

On the contrary, for $p > 2$, for example $p \cong 2.2$, $(p^2 = 5)$, we have

$$\frac{\Delta\omega}{\omega} = -6m_2^2 + 4m_3^2 + \frac{30}{11}m_4^2 + \cdots. \tag{75}$$

The second harmonic reduces the frequency while all higher ones raise it. In such a system the action of the higher harmonics can be compensated by the action of the lower ones.

This circumstance allows one to obtain, with the help of a few filters only, a considerably better frequency constancy than that which

can be expected from the shunt-trapping action. By the suitable detuning of one filter, the compensation of those harmonics which have not been removed by the filters can be reached.

In case the system is beyond the critical state (i.e., there is a certain amount of harmonics), the essential role is played by the derivative of (51), which should be equal to zero.

Differentiating, therefore, expression (51) in regard to the factor x (for instance, the anode voltage etc.), which is responsible for the operative state of the system, we obtain

$$\frac{d}{dx}\left(\frac{\Delta\omega}{\omega}\right) = - \sum_{2}^{\infty}(k^2 - 1)m_k\frac{dm_k}{dx}\cdot \tag{76}$$

The terms dm_k/dx may be positive or negative; the sign depends on the way in which the change of x acts upon the amount of harmonics.

Putting,

$$\frac{d}{dx}\left(\frac{\Delta\omega}{\omega}\right) = 0 \tag{77}$$

we obtain the condition for frequency constancy, notwithstanding the variations of x. In case the signs of dm_k/dx are different, it is always possible to find a working point in the neighborhood of which (76) becomes zero.

If, however, the signs of dm_k/dx are the same, the simple resonant circuit will not allow one to obtain a good frequency constancy. In this case, the scheme represented on Fig. 13 and given by (73) can be useful. It renders it possible to choose the signs and the values of the coefficients of the corresponding terms by choosing p in such a way that the frequency constancy (77) in a certain range of variations of x is obtained.

The similar equalization of the energy may be obtained by the help of a coil N, connected outside of the resonant circuit. In regard to the dynatron oscillator we obtain the following equation:

$$\frac{\Delta\omega}{\omega} = -\frac{1}{2}\sum_{2}^{\infty}\left\{\frac{k^2 - 1}{1 - \frac{N}{L}(k^2 - 1)}\right\}m_k^2. \tag{76}$$

The sign of (76) depends on its denominator and namely, if the order of harmonics

$$k > \sqrt{1 + \frac{L}{N}} \quad \text{then, } L\{\measuredangle\} < 0$$

and if

$$k < \sqrt{1+\frac{L}{N}} \quad \text{then, } L\{\measuredangle\} > 0.$$

Thus, for a given harmonic content N/L can be chosen in such a way that the condition (77) be satisfied. For this purpose the terms of the sum must be positive and negative, which is only possible in the case when N/L is not too great; otherwise the condition (77) would not be fulfilled even for the lowest harmonics (for instance, for $k=2$).

The Experimental Results.

For the verification of the influence of the filters on the frequency constancy, a circuit shown in Fig. 12 was examined. The circuit was excited by means of the dynatron.

In order to obtain always the frequency $f=1000$ cycles at the critical state and to obtain always this critical state for the same value of $V_g=-5$ volts, the capacity C_1 and the resistance S_1 were suitably adjusted. Then V_g was changed to the value $V_g=-2$ volts beyond the critical state, and the frequency variation $\Delta\omega$ was measured. Table I contains the results of measurements for some filters. It shows that the removing of harmonics increases the frequency constancy. With four filters the frequency variation was of the order of 0.1 per cent.

TABLE I

Filter tuned to the harmonics	2nd	3rd	4th	5th	Without filters
$\frac{\Delta\omega}{\omega}$	−3.5	−3.7	−4	−4.7	−5%

Afterwards the influence of the compensation of the circuit was examined. For this purpose four filters were experimentally so tuned that the minimum frequency variation $\Delta\omega/\omega$ for the various operative

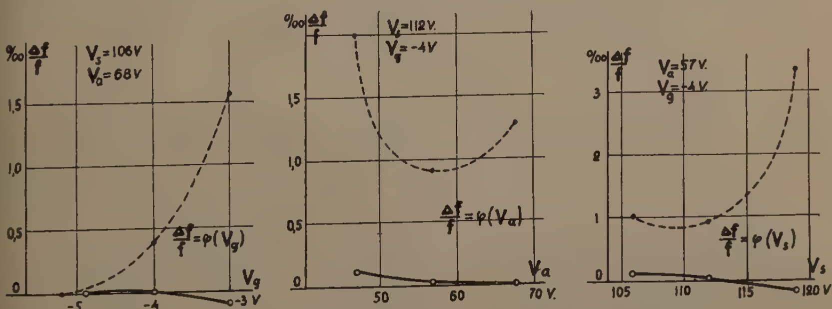

Fig. 14

voltages was obtained. The relation $\Delta\omega/\omega$ as a function of the following factors, grid, anode, and screen voltages, are represented by means of the corresponding curves on Fig. 14. On the same figure are shown the dotted curves for an ordinary circuit (without filters). (The influence of the heating voltage was extremely small in both cases.)

Subsequently, the circuit represented in Fig. 15 was examined. The resistance P was determined experimentally in such a way as to ob-

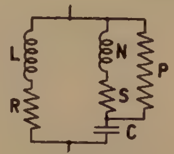

Fig. 15

tain the best frequency constancy for various operative conditions. This value was $P=7000$ ohms. The influence of some factors such as anode and screen voltage are shown in Table II.

TABLE II

V_s	$V_a=66$	76	87	V
110	+0.062	+0.020	—	%
120	−0.073	0	−0.025	%
129	+0.720	−0.196	−0.067	%
V	$V_g=+2V$; $P=7000$ ohms			

Finally (74) was proved quantitatively by means of the circuit represented in Fig. 15, where $L=1$ henry and $N=0.5$ henry (i.e., $p=(1+0.5)/0.5=3$). Indeed, the frequency increased when the grid voltage V_g was changed from the critical value.

Acknowledgment

I wish to thank my assistant, Mr. Z. Jelonek, whose coöperation in some of the measurements and calculations, was very helpful.

Appendix I

$$v = \sum V_k \sin(k\omega t + \alpha_k); \quad i = \sum I_k \sin(k\omega t + \beta_k)$$

$$\frac{dv}{dt} = \sum k\omega V_k \cos(k\omega t + \alpha_k)$$

$$\oint i dv = \oint i\left(\frac{dv}{dt}\right)\cdot dt$$

$$= \sum kV_kI_k \int_0^{2\pi} \sin(k\omega t + \beta_k)\cos(k\omega t + \alpha_k)dt$$

$$+ \sum{}' mV_nI_m \int_0^{2\pi} \sin(m\omega t + \beta_m)\cos(n\omega t + \alpha_n)dt$$

$\sum$ is the sum of the products for $k=1, 2, 3 \cdots \infty$. $\sum'$ is the sum of all possible products for $m=1, 2, 3 \cdots \infty$, $n=1, 2, 3 \cdots \infty$, except $m=n$.

$$\int_0^{2\pi} \sin(k\omega t + \beta_k)\cos(k\omega t + \alpha_k)dt$$

$$= \cos\alpha_k \cdot \sin\beta_k \int_0^{2\pi} \cos^2 k\omega t\, dt - \sin\alpha_k \cos\beta_k \int_0^{2\pi} \sin^2 k\omega t\, dt$$

$$+ (\cos\alpha_k \cos\beta_k - \sin\alpha_k \sin\beta_k) \int_0^{2\pi} \sin k\omega t \cos k\omega t\, dt$$

$$= \pi(\cos\alpha_k \sin\beta_k - \sin\alpha_k \cos\beta_k)$$

$$= \pi \sin(\beta_k - \alpha_k)$$

$$\int_0^{2\pi} \sin(m\omega t + \beta_m)\cdot\cos(n\omega t + \alpha_n)dt = 0$$

Thus:

$$\oint idv = \pi k V_k I_k \sin(\beta_k - \alpha_k)$$

Appendix II

If the phase angle between V_k and I_k is ϕ, we can write $I_k = I_{rk} + jI_{ik}$; the angle between V_k and I_{rk} is 0 degrees and between V_k and I_{ik} is 90 degrees.

The imaginary part of the expression I_kV_k, i.e., $|I_kV_k|_{im} = I_{ik}\cdot V_k$ gives the absolute value $V_kI_k \sin\phi_k$.

Appendix III

Appleton and Greaves[3] have obtained for the voltage on the resonant circuit the expression

$$v = A \sin\omega t + \frac{1}{16}A\left(\frac{1}{2}\frac{\gamma}{\omega}A^2 + \frac{1}{3}\frac{\epsilon}{\omega}A^4\right)\cos\omega t$$

$$+ \frac{1}{6}A\left(\frac{\beta}{\omega}A + \frac{1}{2}\frac{\delta}{\omega}A^3\right)\sin 2\omega t$$

$$-\frac{1}{32}A\left(\frac{\gamma}{\omega}A^{2*}+\frac{3}{4}\frac{\epsilon}{\omega}A^{4}\right)\cos 3\omega t$$

$$+\frac{1}{120}\frac{\delta}{\omega}A^{4}\sin 4\omega t+\frac{1}{384}\frac{\epsilon}{\omega}A^{5}\cos 5\omega t \qquad (1)$$

as well as for the frequency:

$$\omega=\omega_0^2-\frac{1}{12}\left(\beta A+\frac{1}{2}\delta A^{3}\right)^2-\frac{1}{128}\left(\gamma A^{2}+\frac{3}{4}\epsilon A^{4}\right)^2$$

$$-\frac{1}{960}\delta^2A^{6}-\frac{1}{6144}\epsilon^2A^{2}. \qquad (2)$$

From (1) we determine the amplitudes: $V_1 \cong A$

$$V_2=\frac{1}{6}A\left(\frac{\beta}{\omega}A+\frac{1}{2}\frac{\delta}{\omega}A^{3}\right);\quad m_2^2=\frac{1}{36}\left(\frac{\beta}{\omega}A+\frac{1}{2}\frac{\delta}{\omega}A^{3}\right)^2$$

$$V_3=\frac{1}{32}A\left(\frac{\gamma}{\omega}A^{2}+\frac{3}{4}\frac{\epsilon}{\omega}A^{4}\right);\quad m_3^2=\frac{1}{1024}\left(\frac{\gamma}{\omega}A^{2}+\frac{3}{4}\frac{\epsilon}{\omega}A^{4}\right)^2$$

$$V_4=\frac{1}{120}\frac{\delta}{\omega}A^{4};\qquad m_4^2=\frac{1}{(120)^2}A^{6}\frac{\delta^2}{\omega^2}$$

$$V_5=\frac{1}{384}\frac{\epsilon}{\omega}A^{5};\qquad m_5^2=\frac{1}{(384)^2}A^{8}\frac{\epsilon^2}{\omega^2}.$$

The equation (2) we transform as follows:

$$1-\left(\frac{\omega}{\omega_0}\right)^2=2\frac{\Delta\omega}{\omega_0}=-\frac{1}{12}\left(\frac{\beta}{\omega_0}A+\frac{1}{2}\frac{\delta}{\omega_0}A^{3}\right)^2+\cdots.$$

Because, $\omega \cong \omega_0$, we receive, by substituting $m_{2_1} m_3 \cdots$ etc.;

$$\frac{\Delta\omega}{\omega_0}=-\frac{36}{24}m_2^2-\frac{1024}{256}m_3^2-\frac{14400}{1920}m_4\cdots$$

$$=-\frac{1}{2}\left[3m_2^2+8m_3^2+15m_4^2+\cdots\right]$$

i.e., formula (52).

* In their paper, in *Phil. Mag.*, the term A^2 has been here omitted.

Bibliography

1. E. V. Appleton and B. van der Pol, "On the form of free triode vibrations," *Phil. Mag.*, series 6, vol. 42, p. 201; August, (1921).
2. E. V. Appleton and B. van der Pol, "On a type of oscillation hysteresis in a simple triode generator," *Phil. Mag.*, series 6, vol. 43, pp. 177-193; January, (1922).
3. E. V. Appleton and W. M. H. Greaves, "On the solution of the representative differential equation of the triode oscillator," *Phil. Mag.*, series 6, vol. 45, p. 401, (1923).
4. N. W. Mac Lachlan, "On the influence of valve resistance in oscillation generators," *Wireless Engineering and Wireless Engineer*, vol. 9, March, (1932).
5. J. Groszkowski, "The dynatron oscillator. The interdependence of the frequency variation and the content of harmonics," Correspondence, *Wireless Engineer and Experimental Wireless*, vol. 9, September, (1932).
6. I. Wolff, "The alternating-current bridge as a harmonic analyzer," *Jour. Opt. Soc. Amer.*, vol. 15, p. 163, (1927).
7. Y. Rocard, "Sur les oscillateurs à lampe régles près de la limite d'entretion," *Comptes Rendus de l'Acad. des Sciences*, (Paris), tome 194, no. 16, (1932).
8. E. Mallet, "Frequency stabilization of valve oscillators," *Jour. I.E.E.*, (London), vol. 68, p. 578; May, (1930).
9. F. B. Llewellyn, "Constant frequency oscillators," Proc. I.R.E., vol. 19, pp. 2063–2094; December, (1931).
10. R. Gunn, "A new frequency stabilized oscillator system," Proc. I.R.E., vol. 18, pp. 1560–1574; September, (1930).
11. J. B. Dow, "A recent development in vacuum tube oscillator circuits," Proc. I.R.E., vol. 19, pp. 2095–2108; December, (1931).
12. K. E. Edgeworth, "Frequency variation in thermionic generators," *Jour. I.E.E.*, (London), vol. 1, p. 349, (1926).
13. F. M. Colebrook, "The dynatron oscillator," *Experimental Wireless and Wireless Engineer*, vol. 8, p. 581; November, (1931).
14. Y. Kusunose and S. Ishikawa, "Frequency stabilization of radio transmitters," Proc. I.R.E., vol. 20, pp. 310–339; February, (1932).

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

A SIMPLIFIED FREQUENCY DIVIDING CIRCUIT*

By

Victor J. Andrew

(Westinghouse Lamp Co., Bloomfield, N. J.; formerly Doolittle and Falknor, Inc., Chicago, Ill.)

Summary—*A circuit is described in which a type 57, three-grid tube functions as two triodes in two separate oscillator circuits. One of the circuits is a crystal oscillator, and the other is a self-excited circuit which is controlled by the crystal so that it oscillates at a subharmonic of the crystal frequency.*

EQUIPMENT for measuring the frequencies of radio transmitters usually requires a series of harmonics of some low frequency which is itself known with high accuracy. A piezo-electric oscillator is the usual source of stable frequency. However, the harmonic producing oscillator is of a lower frequency than may be economically obtained with a piezo-electric crystal. To overcome this difficulty it is customary to operate the piezo-electric oscillator at a harmonic of the desired frequency, and then by means of a frequency dividing circuit to produce the desired frequency.

The division of frequency is accomplished by operating an oscillator at about the frequency which it is intended to produce by division, and then introducing energy at the control frequency into the oscillator circuit. Under proper conditions, the oscillator frequency will change enough to make one of its harmonics coincide exactly with the control frequency. In order to have the lower frequency oscillator easily controlled, it should be one which is inherently unstable in frequency. The multivibrator has been widely used for the purpose.[1,2] Marrison[3] has pointed out the fact that any tube oscillating on a curved part of its characteristic is suitable for frequency division.

Since the voltage of the control frequency which is introduced into the low-frequency oscillator has a large influence on the "locking-in" of the controlled oscillator,[2] it is usually desirable to have a potentiometer control of this voltage. However, when division by a small integer, say not over four, is desired, "locking-in" is so readily obtained that this control is unnecessary.

* Decimal classification: R357. Original manuscript received by the Institute, March 10, 1933.

[1] J. K. Clapp, "Universal frequency standardization from a single frequency source," *Jour. Opt. Soc. Amer. and Rev. Sci. Instr.*, vol. 15, p. 25; July, (1927).

[2] Victor J. Andrew, "The adjustment of the multivibrator for frequency division," Proc. I.R.E., vol. 19, p. 1911; November, (1931).

[3] W. A. Marrison, "A high precision standard of frequency," Proc. I.R.E., vol. 17, p. 1103; July, (1929).

Fig. 1 shows a simplified frequency dividing circuit which has recently been developed, and which has been found to work excellently when division by a small integer is desired. A crystal oscillator and a frequency divider are both operated with one tube. The tube is a type 57 or 58, three-grid receiving tube. The first or control-grid and the second or screen-grid are used together as the grid and plate of a normal triode. To these elements is connected a piezo-oscillator circuit.

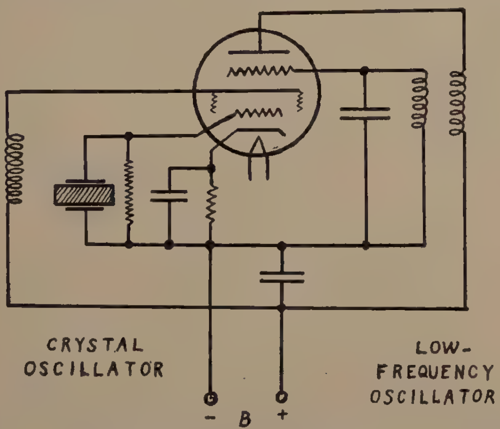

Fig. 1—A piezo-electric oscillator and a frequency divider combined in one tube.

The remaining two elements of the tube, which are the third or suppressor grid and the plate, are also used as the grid and plate of a triode. This second triode is used as a low-frequency, self-excited oscillator. Since the electron current which operates the outer triode has passed through the inner triode, it has been modulated by the frequency of the piezo-electric oscillator. This electron coupling is sufficient to make the low-frequency oscillator "lock-in" at a subharmonic of the crystal frequency.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

AN ANALYSIS OF POWER DETECTION*

By
Rinaldo De Cola
(Consulting Engineer, Cleveland, Ohio)

Summary—*The reasons for overloading of detectors as output devices is discussed. It is shown that in grid circuit rectification replacing the grid leak with a high impedance choke extends the overload point considerably. Using a 247 type pentode 800 milliwatts are obtained at the output circuit with seven per cent maximum distortion.*

EXCEPT for a few isolated instances the use of power detectors working directly into loud speaker systems has been given but little consideration. The main reason for this is apparently the difficulty of obtaining sufficient power output. This limitation in power output is prevalent in both plate circuit or grid circuit detection. Although this overloading is generally ascribed to variations in the operating parameters of the tubes, no definite and conclusive evidence has as yet been given concerning its real cause. Using grid circuit (grid leak and condenser) detection this overloading is generally thought to be caused by the wide fluctuation in R_p values, increasing as the strength of the carrier is increased, because of the gradually increasing negative potential upon the grid from the rectified carrier, and resulting in a decreasing audio output which becomes a minimum as the peak of the carrier is approached. In plate circuit rectification overloading can be definitely traced to the carrier voltage exceeding the grid bias and resulting in grid current. Plate circuit detection however, being very inefficient in power output devices, will not be given further treatment here.

Grid Circuit Detection

Fig. 1 shows a conventional grid circuit rectifier diagram, the values of C and R conforming closely to those suggested by Professor Terman who has done some notable work in this field. By suitably choosing a value for R a point can be located upon the grid-current—grid-voltage characteristic which will result in maximum detection. A modulated carrier impressed upon this grid results in a steady direct-current voltage of negative magnitude which is impressed upon the grid in flowing through resistor R and is due to the rectified carrier. The audio-frequency voltages originally modulating the carrier can be considered in

* Decimal classification: R134. Original manuscript received by the Institute, January 31, 1933.

series with the direct-current potential. The grid potential from the detected carrier serves to depress the plate current, and the audio-frequency voltage is transferred to the plate circuit by virtue of the normal audio amplifying properties of the tube. The magnitude of the transferred audio voltage depends upon the dynamic characteristic of the tube under the condition of the negative potential on the grid. Except where the value of this negative potential is small (weak signals) the characteristics of the tube conform but poorly to those of an audio-frequency amplifier, because of the increased R_p and reduced value of S_m. Also the point on the plate characteristic is no longer upon the straight portion but upon the lower curved portion which results in poor amplification because of the comparatively flat slope and conse-

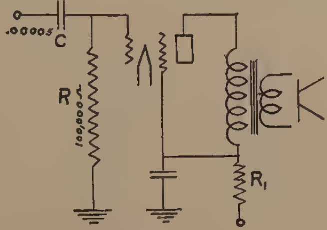

Fig. 1

quent plate rectification which counterbalances the grid rectification. The value of mu for the tube may either increase as in the case of pentodes and screen-grid tubes or remain steady as in three-element tubes. Since when using a pentode tube the negative potential of the grid increases the value of mu which serves to decrease the plate current further, and rectification is accomplished by this dual variation of mu and R_p. Detection by R_p variation only is more generally the case with three-element tubes.

Circuit Considerations

From the above consideration of detection theory it would seem evident that once detection has been accomplished, it would be extremely desirable to adjust the circuit arrangement or voltages once more in order to conform more closely to the conditions of an audio-frequency amplifier. A method which has been suggested is the use of a comparatively large resistance in series with the plate and screen supply in order that when the plate current is decreased on a signal it will be partly compensated for in the lowered potential drop in the load resistance thereby increasing the plate voltage and bringing about more favorable operating conditions from the tube as an audio ampli-

fier. However, unless rather large values of voltages are applied to the system the resistance will not have sufficient control over the plate voltage for optimum operation. This suggestion of a series resistance in the plate circuit brings to mind the advantages of a self-adjusting resistance, a resistance which would under the condition of no-signal have comparatively large resistance in order to protect the tube from abnormal plate current and yet under the condition of an impressed carrier upon the grid would automatically decrease in resistance to a value proportional to the carrier and thus quite accurately and automatically control the plate current and tube characteristics. The use of a small tungsten light bulb was brought to mind by this require-

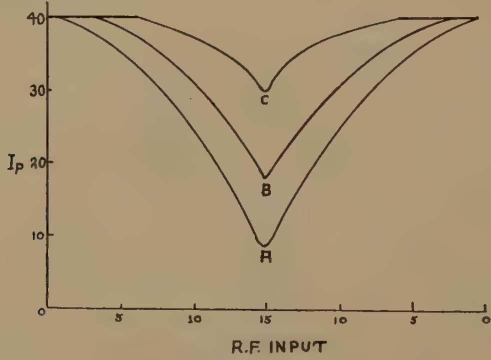

Fig. 2

ment. Measurements on a standard Mazda 7.5-watt 115-volt bulb showed that its resistance at 115 volts (hot) is 1770 ohms (65 milliamperes) and at 1.5 volts only 125 ohms, a change of slightly over fourteen times in resistance from hot to cold. This bulb is rather well adapted to controlling the plate current of power detectors. However under rather strong carriers with perhaps a small amount of modulation the plate voltage could only be properly regulated by the use of several of these bulbs in series and at impressed B voltages in the vicinity of 250 volts for a 233 type pentode. Fig. 2 shows the control of plate current in a 247 pentode as the radio-frequency carrier voltage is varied by tuning the grid input circuit. With no controlling device in the high voltage plate circuit the form of the plate current is shown by curve A, using an ordinary resistance by curve B, and using a single Mazda 7.5-watt 115-volt bulb in series with the B supply by curve C. The superiority of the bulb for control is evident.

Since the use of a Mazda lamp gives a good indication of resonance by the decrease of current which reduces the illumination, this feature

can be used to advantage in indicating resonance in place of a tuning meter.

Referring again to Fig. 1 and the resistance R which by virtue of the grid current from rectification of the carrier in flowing through this resistance to ground builds up the grid bias on this tube resulting in a decrease in plate current and a rather poor operating point for the tube as an audio-frequency amplifier. With detection systems intended only for modulated signal inputs this large change in plate current does not in any way contribute to the audio-frequency output. It would appear desirable to have some load other than a pure resistance in this grid circuit, which would have but little resistance to the normal direct current in this circuit but possess a relatively high impedance for the audio-frequency voltages which it is really intended to amplify. By

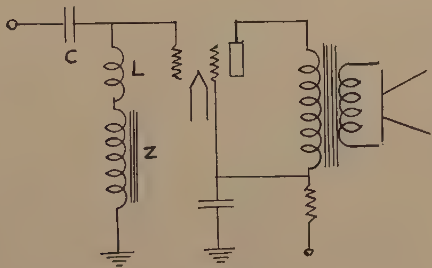

Fig. 3

using a high impedance choke (about 100 henrys) to replace R this result can be accomplished. By utilizing this choke in the grid circuit a carrier impressed upon the input circuit results in but a small voltage being impressed upon the grid because of the low resistance of the choke. This in turn causes no appreciable fluctuation in the dynamic characteristics of the tube, and much better quality and volume can be obtained from the detector without the necessity of subjecting the tube to large normal values of plate current and attempting to protect the device by means of the automatically adjusting resistance as suggested above.

In order to locate the optimum detection point on the $I_g - E_g$ curve a small positive potential can be impressed upon the grid in series with the ground end of the choke; by making this potential variable the best detection point can be quickly located without any difficulty.

A circuit diagram of this detection system is shown in Fig. 3. The choke L is used to prevent radio-frequency leakage to ground through the distributed capacity of the impedance Z.

Available Power Output

In Fig. 4 are shown several curves showing the available output from the various systems discussed. These tests were run entirely on pentodes and confined to types 247 and 233. The radio-frequency signal used was 100 per cent modulated at 400 cycles, and the maximum permissible distortion (second harmonic) seven per cent.

Curve *A* was taken with a 247 using a Mazda 7.5-watt lamp in series with the 300-volt power supply. Normal plate current with no signal was about 50 milliamperes. Using a grid condenser (0.00005 microfarad) and a grid resistance (100,000 ohms) the maximum output was about 620 milliwatts with 15 volts of radio frequency applied to the input.

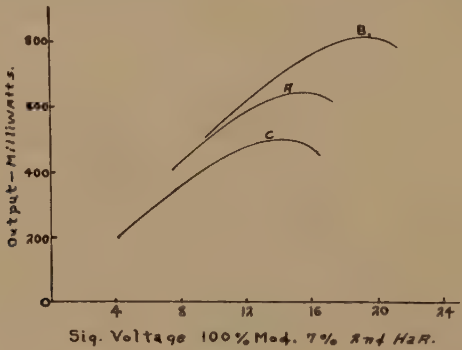

Fig. 4

Curve *B* shows the same tube but with a 150-henry choke replacing the resistance *R*, the value of *C* remaining the same, the Mazda lamp being removed and the plate voltage reduced to a value sufficient to give a total plate and screen current of 50 milliamperes. Although the sensitivity of this arrangement was not as good as curve *A* the maximum realizable power was somewhat greater, being about 800 milliwatts.

Curve *C* was obtained by using two 233 type pentodes in parallel, the maximum normal plate and screen currents for both tubes being 50 milliamperes. An impedance *Z* was used in place of *R*, *C* being kept at 0.00005 microfarad. Here with an impressed carrier of 13 volts, 500 milliwatts were obtained from the tubes before overloading became serious.

General

Although the results above have been obtained by running tubes at space currents somewhat higher than the values recommended by manufacturers it is felt that this should not seriously limit the life of

the tube if used in this fashion continuously, the only limitation being the possibility of ionization due to the higher plate current. However, if the tubes have been carefully exhausted this should not be a serious difficulty in view of the total plate dissipation being but slightly higher than that specified by manufacturers.

In terms of present-day audio outputs for the better grades of receivers the magnitudes of power output as mentioned in this paper may appear rather insufficient. It must be appreciated, nevertheless, that in small midgets or perhaps supermidgets this amount of power output is probably more than the available baffling area can utilize and certainly much more than the average home can generally use.

The use of power detectors of this type can supply sufficient power to a class B amplifier without the necessity of an intermediate driver tube.

Proceedings of The Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

MODES OF VIBRATION OF PIEZO-ELECTRIC CRYSTALS*

By

N. H. Williams

(University of Michigan, Ann Arbor, Michigan)

Summary—(*a*) *A crystal is made to oscillate by subjecting it to the action of an air wave produced at a considerable distance from the crystal by a jet of air escaping from a small tube. The piezo-electric charge developed on the surface of the crystal as a result of the oscillation is mapped out by means of a tuned amplifier, and the vibration is analyzed into a fundamental and many overtones. For crystals that are long in one dimension, the overtones are nearly exact harmonics.*

(*b*) *Exciting the crystal electrically by means of two tubes and especially designed electrodes, any harmonic up to the tenth can be produced. The crystal usually has only one mode of vibration for each pair of electrodes.*

Introduction

When a crystal is vibrating in a piezo-electric oscillator a very large number of harmonics may be detected, any one of which may be amplified and brought into prominence. This is perfectly well known, and probably all would agree that these harmonics are produced in the circuits and do not represent different modes of vibration of the crystal. The present paper does not deal with these harmonics but is concerned with actual crystal vibrations.

With square or round plates a considerable number of overtones may be produced, and it is found that they seem to be entirely unrelated to each other and to the fundamental. Wright and Stuart[1] have reported as many as a dozen different frequencies from a circular plate and at least eight frequencies from a square plate; and the list of frequencies shows no resemblance to a series of harmonics. The most satisfactory method described by them for making these studies is as follows. Lycopodium powder was sprinkled over the face of the vibrating crystal; this collected at the nodal lines and the pattern which it formed was photographed. A large number of these photographs are reproduced in the paper. The patterns are so complicated that it becomes evident that one ought not to expect to find a simple relation among the different frequencies. Colwell[2] has reproduced photographs of sand patterns on square and round Chladni plates and has pointed

* Decimal classification: 537.65 × R214. Original manuscript received by the Institute, December 22, 1932.

[1] *Bureau of Standards Journal of Research*, vol. 7, no. 3, p. 519; September, (1931).

[2] Proc. I.R.E., "The vibration of quartz plates," vol. 20, no. 5, p. 808; May, (1932).

out the similarity between these patterns and those photographed by Wright and Stuart.

In the case of long metallic rods the frequencies of the longitudinal vibration approach exact multiples of the fundamental frequency. A similar relation should be found if quartz plates are cut so that one dimension is much larger than the others.

Mechanical Excitation of a Crystal

The analogy between the vibrations of a long rod and those of an open organ pipe is apparent. The organ pipe produces a fundamental note whose wavelength is approximately twice the length of the pipe, and in addition a series of overtones which are nearly exact harmonics. In order to find out if a quartz crystal, mechanically ex-

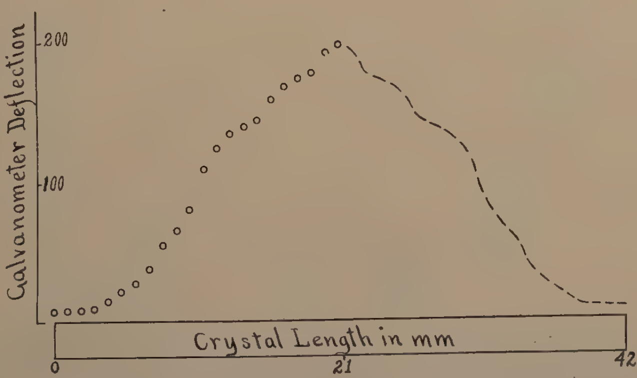

Fig. 1—Crystal mechanically excited. Amplifier tuned to the fundamental frequency of the crystal. Ordinates indicate root-mean-square values of potential at different distances from the end of the crystal. Experimental points determined for half the crystal, the second half of the curve being inferred from the first.

cited, would show a complete series of harmonics, the following experiments were performed. The crystals used were 43, 68, and 100 millimeters long, respectively, each cut with its longest dimension perpendicular to the electric axis. The other dimensions of the 100 millimeter crystal were 10 and 20 millimeters, respectively. The crystal was laid on a metal plate, which was connected to the grounded shielding box of an amplifier. The input wire of the amplifier was connected to a very narrow electrode that could be moved by a micrometer screw across the face of the crystal from end to end. This electrode was surrounded by a grounded guard plate so that it was protected from influences other than that of the voltage developed on the particular part of the crystal that was directly under the electrode.

To excite the crystal mechanically and set it in vibration, compressed air was allowed to escape from a nozzle placed at a distance of six feet from the crystal. This made a faint hissing sound, and evidently produced a considerable volume of supersonic waves of random frequency. These air waves, striking the end of the crystal, set it in motion with a longitudinal vibration. At a place in the crystal where condensations and rarefactions are at their maxima, i.e., at a node, an alternating electrical potential is set up, which is communicated by the movable electrode to the amplifier, and a deflection of the output galvanometer is produced. As the electrode was moved across the face of the crystal, the mean square voltage could be measured and plotted

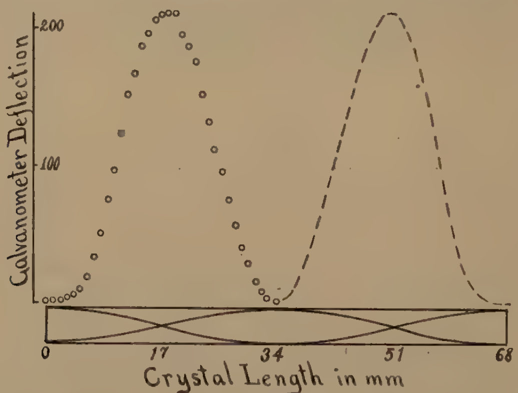

Fig. 2—Crystal mechanically excited. Amplifier tuned to the frequency of the second harmonic of the crystal. Experimental points determined for half the curve.

against the distance along the crystal measured from one end. Fig. 1 shows such a graph, plotted when the amplifier was tuned to the fundamental frequency of the crystal. Only half the curve was actually obtained experimentally, the second half being a duplicate of the first. The amplifier was sharply tuned and its output was zero unless it was tuned to some natural frequency of the crystal. When it was tuned to a frequency double that of the fundamental, the curve of Fig. 2 was obtained. By varying the tuning of the amplifier it was found that the first seven harmonics could be observed. In the last case seven clearly defined maxima were found, with the galvanometer deflection falling practically to zero between adjacent maxima. The only change made in obtaining these different harmonics was in the tuning of the amplifier, hence it is evident that the crystal was executing vibrations in all these modes at the same time. This is analogous to the behavior of the open organ pipe, which produces the complete series of harmonics. The

curves for the even harmonics are smooth, since the amplifier responds only to the one to which it is tuned. When tuned to an odd harmonic, the amplifier is slightly sensitive to other odd harmonics of a higher order. The effect of at least one higher harmonic is seen as a ripple superposed upon the main curve. These curves are not reproduced here, since the ripples are not sufficiently definite to furnish much information. However, the maxima and minima in all cases are clearly defined, and the galvanometer deflections are large.

Electrical Excitation of the Various Harmonics

In the organ pipe adjacent nodes are in opposite phase; i.e., at the instant of maximum compression at any node, there is a maximum rarefaction at the adjacent nodes on either side. In the crystal oscillating

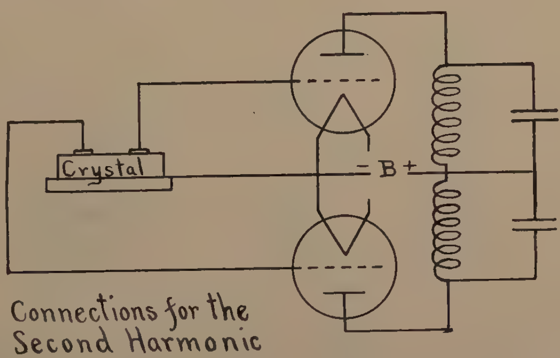

Fig. 3—Push-pull circuit for causing the crystal to vibrate at the frequency of its second harmonic.

at one of its higher harmonics, the same condition would exist, and, as a result, adjacent nodes would show electrification in opposite phase. This suggests the possibility of making the crystal oscillate at the frequency of any one of its harmonics by means of a push-pull scheme shown in Fig. 3. This figure illustrates the connections for producing the second harmonic. For this mode of vibration, there is an antinode at the middle and another at each end of the crystal, while the nodes appear in opposite phase a quarter of the crystal length from each end. The electrodes in contact with the crystal are placed over the nodes and connected one to each of the grids of the tubes. The plate upon which the crystal lies is connected to the mid-point between the filaments. With this arrangement the crystal oscillates at the frequency of its second harmonic and in no other way. Push-pull circuits of a somewhat different character have been used by others, notably by

Harrison,[3] who states that some of his circuits can be used for exciting any of the different modes of vibration.

If the crystal is to be made to oscillate at the frequency of its third harmonic, there must be three nodes, one at the middle and one at a sixth of the crystal length from each end. The nodes near the ends are in the same phase but in a phase opposite to that of the middle node. In this case one electrode is cut to cover both end nodes, and the other is simply a rectangular piece of metal placed at the middle of the crystal. In Fig. 4 the upper right-hand diagram shows the shape of the electrodes for this case and the way they are placed on the upper surface of the crystal. The different diagrams in this figure give the

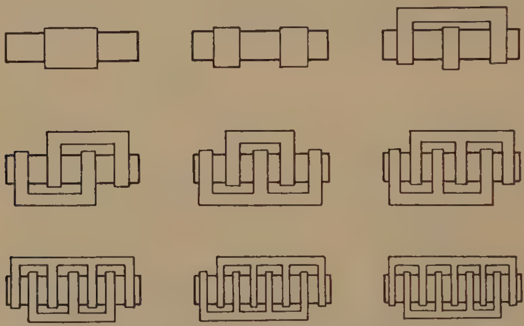

Fig. 4—Forms of electrodes and their positions on the crystal for producing the first nine harmonics.

shapes and the positions of the electrodes for producing the first nine harmonics.[4] The tenth harmonic has nearly the same frequency as that of the vibration through the shortest dimension of the crystal, so that it was obscured by this other mode of vibration. However, the first nine harmonics were easily obtained, the oscillations were vigorous, and their frequencies were easily measured. The crystal showed only one mode of vibration for each pair of electrodes, and that had a frequency so nearly an exact multiple of the fundamental that the author has taken the liberty of calling these overtones harmonics.

[3] Proc. I.R.E., "Push-pull piezo-electric oscillator circuits," vol. 18, no. 1, p. 95; January, (1930).

[4] It is probable that W. G. Cady's patent, No. 1,782,117 of November, 1930, covers the use of electrodes cut as here described. The title of the patent is "A method of stimulating a piezo-electric body to vibrate at a frequency higher than the fundamental frequency of said body, which comprises subjecting only a fractional portion to an alternating electric field of suitable frequency."

Following is a list of the harmonics studied and the frequencies that were measured. The last column gives the frequency divided by the number of the harmonic, which should be the frequency of the fundamental if the overtones had been exact harmonics.

Number of the Harmonics	*Frequency, Cycles per Second*	*Frequency Divided by Number*
First	26,400	26,400
Second	52,500	26,250
Third	79,000	26,330
Fourth	101,200	25,300
Fifth	126,300	25,260
Sixth	164,000	27,300
Seventh	187,000	26,700
Eighth	206,000	25,750
Ninth	240,000	26,600

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

PROPAGATION OF WAVES OF 150 TO 2000 KILOCYCLES PER SECOND (2000 TO 150 METERS) AT DISTANCES BETWEEN 50 AND 2000 KILOMETERS*

By

BALTH. VAN DER POL (Eindhoven, Holland); T. L. ECKERSLEY (Chelmsford, England); J. H. DELLINGER (Washington, U. S. A.); and P. LE CORBEILLER (Paris, France).

DURING the course of the Madrid International Radio Conference (September to December, 1932) it was felt that it would be of value to possess a condensed statement of the propagation characteristics of radio waves, for the ranges of frequency and distance quoted in the title. Colonel Angwin, chairman of Subcommittee 1 of the Technical Committee of the Conference, entrusted four members with the task of preparing such a statement, which was duly prepared and was presented to the Conference on October 11th, 1932.

It was thought that this condensed information might also be of interest to a broader technical public, and it is for that reason presented here. The information is in the form of field intensities produced at distances between 50 and 2000 kilometers in the above frequency range, for both day and night.

The data are presented in eight graph sheets (Figs. 1 to 8), Figs. 1 to 4 being for transmission over land having the average conductivity of 10^{-13} C. G. S. electromagnetic units, and Figs. 5 to 8 being for transmission over sea (conductivity 10^{-11}). It is important to note that land conductivities vary greatly. The value used here is an average of the results of experience in Europe and America. For land of greater conductivity, the data are nearer those given for sea transmission, and vice versa.

Each of the figures shows root-mean-square field intensities in millivolts per meter for one kilowatt of radiated power, as a function of distance in kilometers, both for day and for night transmission. For other values of radiated power, the field intensities are proportional to the square root of the power. The night data represent conditions from about sunset to midnight. As each figure plainly shows, for short distances the night and day values are the same. At greater distances the night values exceed the day values and are very variable. The night

* Decimal classification: R113.7. Original manuscript received by the Institute, December 27, 1932.

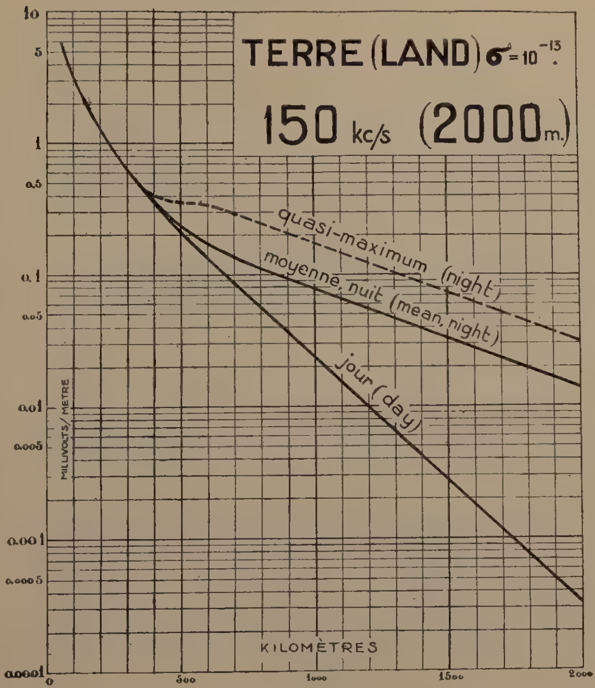
TERRE (LAND) σ=10⁻¹³
150 kc/s (2000 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/METRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 1

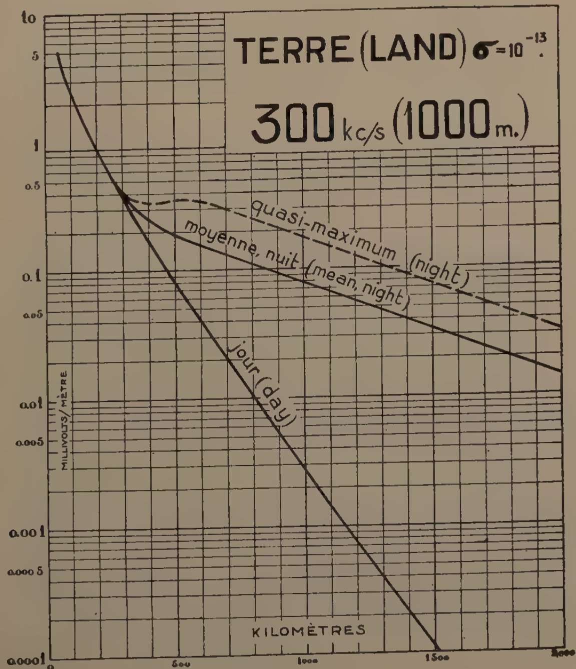
TERRE (LAND) σ=10⁻¹³
300 kc/s (1000 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 2

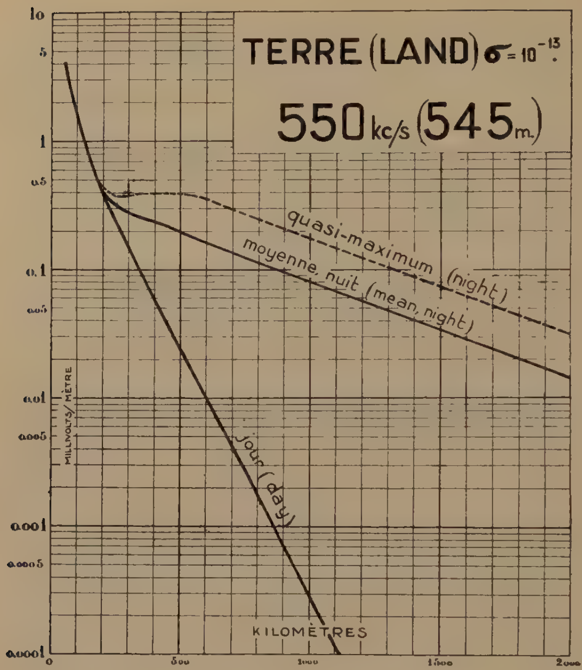
TERRE (LAND) σ = 10⁻¹³.
550 kc/s (545 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS / MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 3

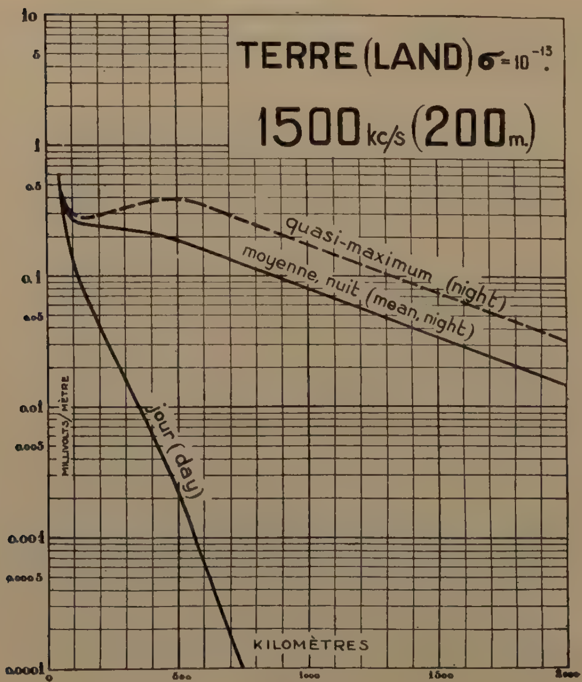
TERRE (LAND) σ = 10⁻¹³.
1500 kc/s (200 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS / MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 4

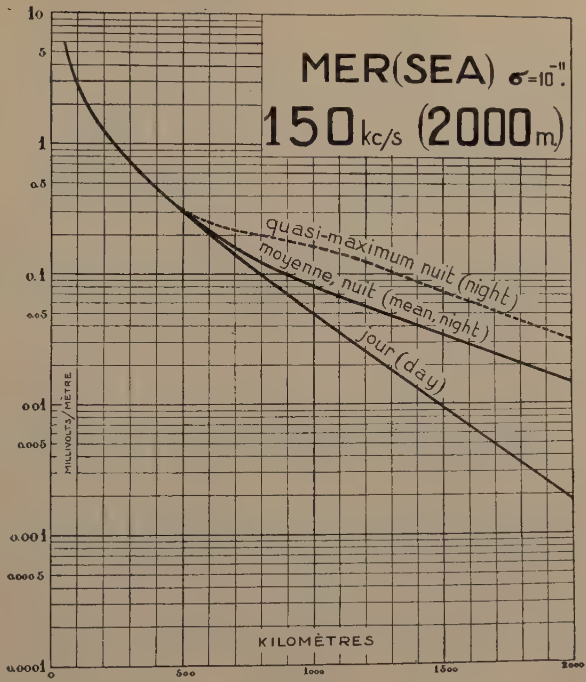
MER(SEA) σ=10⁻¹¹.
150 kc/s (2000 m)
quasi-maximum nuit (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 5

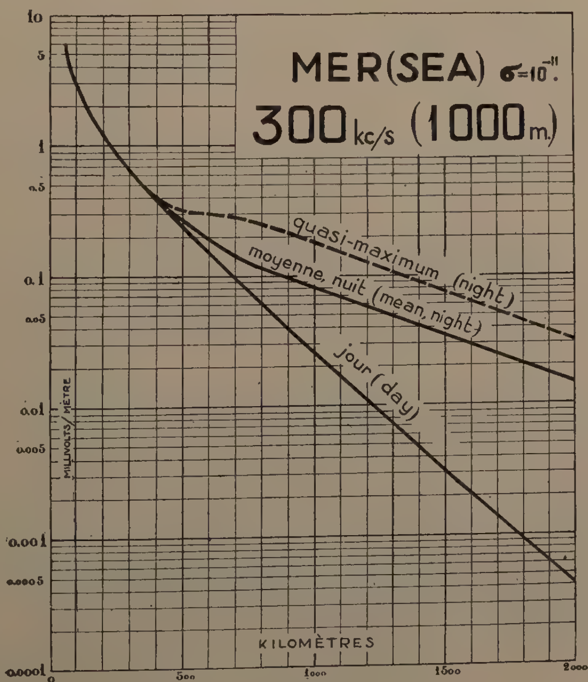
MER(SEA) σ=10⁻¹¹.
300 kc/s (1000 m)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 6

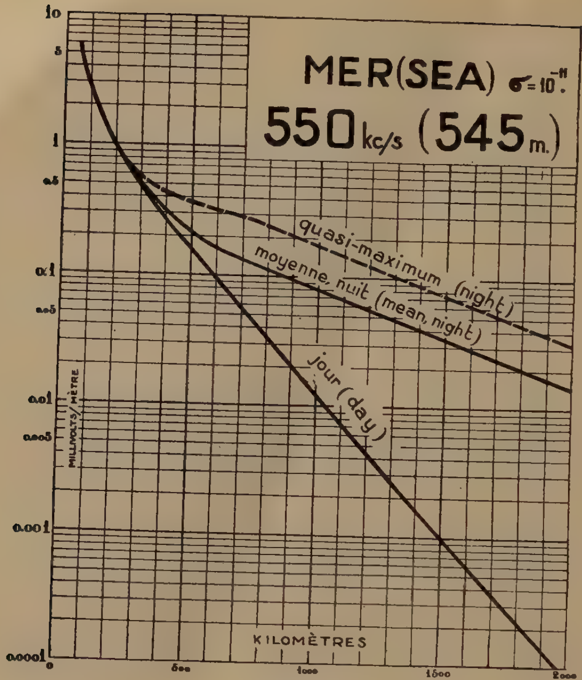
MER(SEA) σ=10⁻¹¹
550 kc/s (545 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0001
0
500
1000
1500
2000

Fig. 7

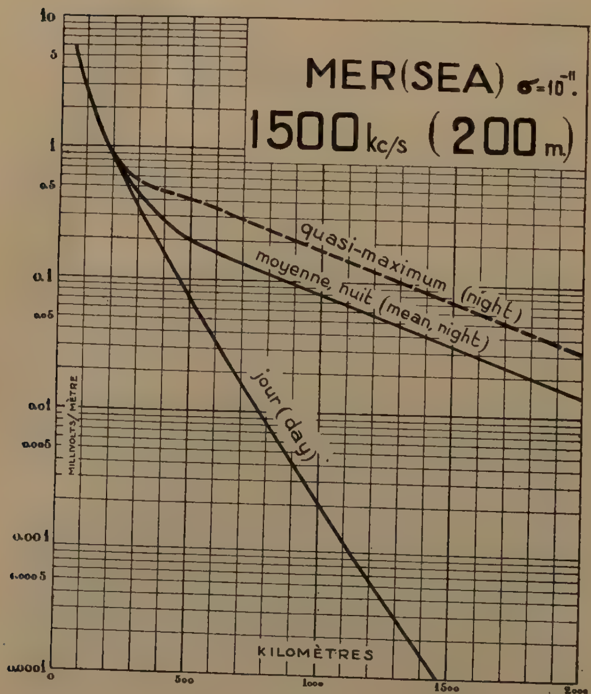
MER(SEA) σ=10⁻¹¹
1500 kc/s (200 m.)
quasi-maximum (night)
moyenne, nuit (mean, night)
jour (day)
MILLIVOLTS/MÈTRE
KILOMÈTRES
10
5
1
0.5
0.1
0.05
0.01
0.005
0.001
0.0005
0.0001
0
500
1000
1500
2000

Fig. 8

values given in the figures are average and quasi-maximum values, defined respectively as those which are exceeded by the instantaneous values 50 per cent and 5 per cent of the time. The quasi-maximum values are shown rather than the absolute maxima because the latter occur seldom and moreover depend upon the length of time over which the observations are taken.

The data are given for four frequencies. Figs. 1 and 5 are for 150 kc/s (2000 m), Figs. 2 and 6 for 300 kc/s (1000 m), Figs. 3 and 7 for 550 kc/s (545 m), Figs. 4 and 8 for 1500 kc/s (200 m).

The graphs show that, at distances from about 500 to 2000 kilometers, the night values are the same for all frequencies involved and are the same for sea and land, whereas the day values are different for each of these conditions.

It must be emphasized that the data presented here are merely average data; actual cases vary from about half to twice the values given for the lower frequencies, and from about one third to three times for the higher frequencies. The differences in practice are due to differences in ground conductivity, irregularities in terrain (e.g., hills, forests, towns), antenna characteristics, and conditions in the ionosphere (ionized region of the upper atmosphere).

The graphs are based on experimental data from many sources. When the nature of the several observations and the character of the phenomena are considered, the concordance of the data is quite satisfactory. The labor of many workers for years past has established the day values shown. For the night values the observational data available were plotted and a curve drawn through them with the result shown in the figures.

Bibliography

1. A. Sommerfeld, "Uber die Ausbreitung der Wellen in der drahtlosen Telegraphie," *Ann. der Physik*, 4th ser., vol. 28, no. 4, pp. 665–736, (1909).
2. G. N. Watson, "Transmission of electric waves round the earth," *Proc. Roy. Soc.* (London), ser. A, vol. 95, no. A673, pp. 546–563; July 15, (1919).
3. Balth. van der Pol, "On the propagation of electromagnetic waves round the earth," *Phil. Mag.*, 6th ser., vol. 38, no. 225, pp. 365–380; September, (1919).
4. A. Sommerfeld, "Uber die Ausbreitung der Wellen in der drahtlosen Telegraphie," *Ann. der Physik*, 4th ser., vol. 81, no. 25, pp. 1135–1153; (1926).
5. P. P. Eckersley, T. L. Eckersley, and H. L. Kirke, "Design of transmitting aerials for broadcasting stations," *Jour. I.E.E.* (London), vol. 67, pp. 507–520; April, (1929); discussion, pp. 520–526.
6. Bruno Rolf, "Numerical discussion of Professor Sommerfeld's attenuation formulae for radio waves," *Ingeniorsvetenskapsakademiens Handlingar*, no. 96, (1929).
"Graphs to Professor Sommerfeld's attenuation formula for radio waves," Proc. I.R.E., vol. 18, no. 3, pp. 391–402; March, (1930).
7. Balth. van der Pol, "Uber die Ausbreitung electromagnetischer Wellen," *Zeit. für Hochfrequenz.* vol. 37, no. 4, pp. 152–156; April, (1931).

8. Comite Consultatif International Technique des Communications Radioelectriques. Deuxieme reunion, Copenhague, Mai-Juin, 1931. Documents, pp. 367–461.
9. Convention Radiotelegraphique Internationale. Propositions pour la Conference Radiotelegraphique Internationale de Madrid (1932). Supplement no. 11, June 1, (1932).
10. Federal Radio Commission. Fifth annual report to the Congress of the United States for the fiscal year 1931.
11. T. L. Eckersley, "On the connection between the ray theory of electric waves and dynamics," *Proc. Roy. Soc.* (London), ser. A, vol. 132, no. A819, pp. 83–98; July, (1931).
12. T. L. Eckersley, "Radio transmission problems treated by phase integral methods," *Proc. Roy. Soc.* (London), ser. A, vol. 136, no. A830, pp. 499–527; June, (1932).
13. T. L. Eckersley, "Long wave transmission treated by phase integral methods," *Proc. Roy. Soc.* (London), ser. A, vol. 137, no. A831, pp. 158–173; July, (1932).
14. Convention Radiotelegraphique Internationale. Propositions pour la Conference Radiotelegraphique Internationale de Madrid (1932). Supplement no. 14, July 1, (1932).
15. Union Internationale de Radiodiffusion. Memorandum no. 1, (1932).
16. Union Internationale de Radiodiffusion. Maxima et minima horaires du rayon indirect des stations mesurees. March, (1932).
17. U. S. Bureau of Standards, Distance ranges of radio waves. Letter circular no. 317, January 25, (1932).
18. S. S. Kirby and K. A. Norton, "Field-intensity measurements at frequencies from 285 to 5,400 kilocycles per second." U. S. Bureau of Standards, Research paper no. 429, (1932). *Bureau of Standards Journal of Research*, vol. 8, pp. 463–479; January–June, (1932).
19. J. Bion and P. David, "Sur l'affaiblissement des ondes moyennes et intermediaires se propageant de jour sur mer," *Comptes Rendus*, vol. 194, no. 20, pp. 1723–1724; May 17, (1932).
20. T. L. Eckersley, "Direct-ray broadcast transmission." Proc. I.R.E., vol. 20, no. 10, pp. 1555–1579; October, (1932).
21. Unpublished observations.
    (1) Royal Canadian Corps of Signals (Canada).
    (2) British Broadcasting Corporation (Great Britain).
    (3) N. V. Philips Radio, Physical Laboratory (Holland).
    (4) U. S. S. R. Delegation.
    (5) Navy Department (U. S. A.).
    (6) Research Department, Marconi Company (Great Britain).
    (7) Navy Department (France).
    (8) R. Braillard (U. I. R.)

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

A METHOD OF CALCULATION OF FIELD STRENGTHS IN HIGH-FREQUENCY RADIO TRANSMISSION*

By

SHOGO NAMBA AND TARO TSUKADA
(Electrotechnical Laboratory, Ministry of Communications, Tokyo, Japan)

Summary—*The theory on the transmission of high-frequency radio waves developed by one of the authors in a previous paper*[1] *is applied to practical problems, and a systematic method of calculating the field strengths of high-frequency waves is established.*

The property of the ionization chart is explained and the method of construction of the chart is given.

Practical examples of calculations are shown with respect to Tokio—Cape Town and Tokio—Melbourne circuits. The present method is most useful not only for the projection of a new high-frequency communication route, but also for the investigations of high-frequency transmission phenomena.

From a number of measurements made on high-frequency transmission, it is shown that the coefficient of recombination in the F layer is about 1.5×10^{-10} [sec^{-1}] *this value being in fairly good coincidence with the theoretical value.*

LIST OF SYMBOLS

(Arranged in the alphabetical order, excluding those already given in the first part of the previous paper.[1])

d = distance in kilometers between transmitter and receiver.
e = charge of an electron, or base of natural logarithm.
f_0 = limiting frequency.
h = altitude of the sun.
I = rate of ion production [$\sec^{-1}$, cm^{-3}]
I_0 = maximum value of I.
$\mathfrak{N}$ = value of the electron density at the time of the cessation of ionization.
$\mathfrak{N}'$ = value of the electron density at the virtual time of the cessation of ionization.
W = radiated power in kilowatts.
$Z_{\max}$ = the height at which I becomes maximum in the case of $\chi=0$.
δ = north declination of the sun.
θ = north latitude.
ρ = density of the constituent of the upper atmosphere.
ϕ = local hour measured from the noon.
χ = zenith distance of the sun $=\dfrac{\pi}{2}-h$.

* Decimal classification: R270. Original manuscript received by the Institute, December 5, 1932. Abbreviated translation from the original paper in Japanese, *J.I.E.E.* (Japan), vol. 52, no. 11; November, (1932); *Researches of the Electrotechnical Laboratory*, no. 336, (1932).

I. Introduction

Although recent developments in short-wave technique have brought to light a number of unknown phenomena, we as yet know very little about the method of calculating the field intensity in high-frequency transmission. As for low-frequency waves there have already been published various kinds of experimental formulas by Austin, Cohen, Espenschied, Eckersley, etc., making contributions toward the projection of a new communication circuit. For high-frequency waves the transmission phenomena are so complicated, contrary to those for low-frequency waves, that we have not been able to devise any reliable method of calculation.

During the past several years, the authors have made a study of the high-frequency transmission theory, and have made at the same time a long series of observations of high-frequency field strengths.[2] From the results of these theoretical and experimental investigations, the authors have developed a new method of calculating the high-frequency field strengths, by means of which the field strengths at the receiving point can be determined in terms of microvolts per meter, when the transmission distance, radiated power, design of the aerial, etc., are known.

The proposed method provides a quantitative relation between the output of a transmitter, number of dipole elements of beam aerials for transmission and reception, and frequency of a wave. It is particularly useful in projecting a new high-frequency communication route. Discrepancies between observed and calculated values of field strengths do not ordinarily exceed five decibels in our experience.

II. Résumé of the Theory on High-Frequency Transmission

The authors have treated in the previous paper[1] a general theory on the propagation of radio waves, and here some important parts of the theory will again be taken up for the sake of reference in the further treatment.

It has been pointed out that the propagation of high-frequency waves in the upper ionized medium can be treated by the law of geometrical optics. The wave penetrates through the ionized layer or is bent down therefrom according to the relative amount of the maximum electron density of the layer N_{max} compared to the value of N_0, which is defined by

[1] Numbers refer to Bibliography.

$$N_0 = \frac{m\omega^2}{4\pi e^2}\cdot\left[\cos^2 i_0 + \frac{2Z}{r_0}\right]. \tag{1}$$

Since the ionized medium is composed of two layers, the E and F layers, respectively, there are the following five possible arrangements of wave paths, as shown in Fig. 1, according to the relations between N_0 and N_{max}. Path A corresponds to the case in which $N_0 < N_{E\,max}$, path B to the case $N_0 = N_{E\,max}$, paths C, D, and E to the cases $N_{F\,max} > N_0 > N_{E\,max}$, $N_0 = N_{F\,max}$, $N_0 > N_{F\,max}$, respectively. Path C represents a trajectory generally realized under the normal condition of high-frequency transmission.

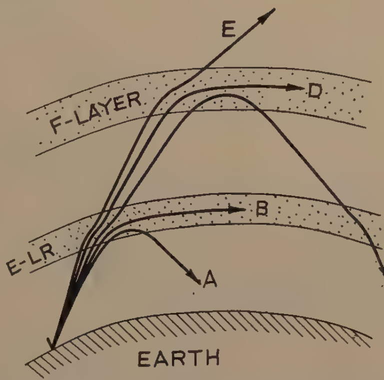

Fig. 1—Possible arrangements of wave paths in a medium composed of two ionized layers.

Next, regarding the attenuation, we have previously classified it into two kinds; the *attenuation of the first kind* and the *attenuation of the second kind.* The former is shown by

$$\Gamma_1 = \frac{2\pi e^2}{mc\omega^2}\cdot\frac{1}{\sqrt{\cos^2 i_0 + \dfrac{2Z_E}{r_0}}}\cdot\int_{-\infty}^{+\infty} N\cdot v\cdot dZ \tag{2}$$

and is characterized by having a property that is, roughly speaking, inversely proportional to ω^2 and $\cos i_0$. The second kind attenuation may be expressed by the following equation,

$$\Gamma_2 = \frac{m\omega^2}{4\pi ce^2}\cdot\left[\cos^2 i_0 + \frac{2Z}{r_0}\right]^{3/2}\cdot\int_0^1 \frac{v\cdot\xi\cdot d\xi}{\dfrac{dN}{dZ}\sqrt{1-\xi}} \tag{3}$$

where,

$$\xi = \frac{4\pi Ne^2}{m\omega^2} \cdot \frac{\left[1 + \frac{Z}{r_0}\right]^2}{\cos^2 i_0 + \frac{2Z}{r_0}}.$$

As shown in (3), Γ_2 is known to be directly proportional, contrary to Γ_1, to a certain power of ω and $\cos i_0$.

In practical cases of high-frequency transmission, the attenuation of the first kind, from which the wave suffers in the E layer, is much more important than that of the second kind from which the wave suffers in the F layer. Therefore, the necessary conditions for a satisfactory transmission of high-frequency waves can be summarized into the following two items:

(a) The condition $N_0 < N_{F\max}$ should always be satisfied.

(b) Attenuation in the E layer should be as small as possible.

These two conditions, however, are inconsistent with each other, because the ionizing agents for the E and F layers must both be closely related to solar phenomena. From a practical point of view, it is generally recognized that daylight transmission is limited by the attenuation caused in the E layer, while for night transmission the limiting factor is not the *attenuation* in the E layer, but the *electron limitation* in the F layer.

III. Ionization in the Upper Atmosphere

Assuming that the radiation from the sun is to be of monochromatic nature, and also that the atmospheric pressure changes in accordance with an exponential law with respect to the height above the surface of the earth, Chapman[4,5] has shown that the rate of ion production at a point in the earth's atmosphere may be represented by the following equation,

$$I = I_0 e^{[1-\zeta-e^{-\zeta}\cdot f(\chi)]} \tag{4}$$

where, $f(\chi)$ and ζ represent the following functions:

$$f(\chi) = \sec\chi \qquad \text{for } \chi < 75^\circ \tag{5}$$

$$f(\chi) = \sqrt{\frac{\pi}{2}\left(\frac{r_0 + Z}{H}\right)} \qquad \text{for } \chi \geqq 90^\circ \tag{6}$$

$$\zeta = \frac{Z - Z_{\max}}{H} \tag{7}$$

and,

$$H = \frac{kT}{Mg} \tag{8}$$

T = absolute temperature,
M = molecular mass to be ionized,
g = gravity acceleration,
k = constant,
χ = zenith distance of the sun (as shown in Fig. 2),
I = rate of ion production,
I_0 = maximum value of I
Z_{max} = height at which I becomes maximum when $\chi = 0$.

In our present case, however, the necessary quantity is not I but N, the electron density of the ionized layer. We shall next consider how the value of N varies in a day.

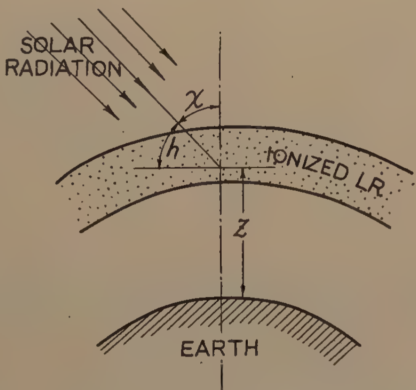

Fig. 2—Diagram illustrating variables involved in computation of ionization.

It may generally be recognized that at a height of more than 100 kilometers above the ground the number of negative ions is much less than that of electrons. Denoting the coefficient of recombination between an electron and a positive ion by α, there exists the following well-known equation:

$$\frac{dN}{dt} = I - \alpha N^2. \tag{9}$$

Since I must be zero during nighttime, (9) may be solved as shown by the following expression,

$$\alpha t = \frac{1}{N} - \frac{1}{\mathfrak{N}} \tag{10}$$

where $\mathfrak{N}$ represents the value of N at the instant when the ionization has just ceased and t the time after the cessation of the ionization.

(*a*) *Electron density in the F layer*

Assume that the ionization is suddenly stopped at a particular value of χ, say χ_0, and let $\mathfrak{N}_F'$ be the virtual electron density of the F layer at the same instant. Although the change of the electron density follows the full-line curve in Fig. 3, yet it may safely be assumed that

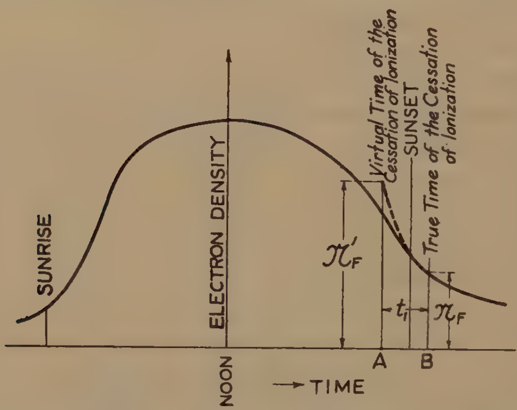

Fig. 3—Illustration of assumed variation in electron density.

the variation of N follows the dotted-line curve in the same figure, so far as the matter under discussion is confined to the nocturnal variation of N. The interval t_1 between the actual and the virtual time of the cessation of the ionization may be shown by,

$$\alpha_F t_1 = \frac{1}{\mathfrak{N}_F} - \frac{1}{\mathfrak{N}_F'} \tag{11}$$

where the quantities with the suffix F are confined to the F layer.

Estimations will be made of $\mathfrak{N}_F'$ and t_1 below. Generally, in a region of lower latitudes, $\mathfrak{N}_F$ is greater as the intensity of the sunshine is stronger, but the value of t_1 is smaller. While in a region of higher latitudes $\mathfrak{N}_F$ is smaller but t_1 is greater. With regard to the season, $\mathfrak{N}_F$ is great but t_1 is small in summer. These two opposite properties tend to compensate each other, and we arrive at an important conclusion that *the value of $\mathfrak{N}_F'$ may roughly be assumed to be constant throughout any season of a year and latitude.*

The maximum electron density $N_{F\max}$ may be obtained by putting $dN/dt=0$ in (9), then we have,

$$N_{F\max} = \sqrt{\frac{I_{0F}}{\alpha_F}} . \tag{12}$$

Assuming $N_{F\max}=2\times10^6$ and $\alpha_F=1.5\times10^{-10}$ [$\sec^{-1}$], the maximum electron density is known to exist at a height of about 280 kilometers above the ground. If we further assume $\chi_0=85$ degrees, we obtain

$$\mathfrak{N}_F' \fallingdotseq 9 \times 10^5$$

and the value is known to be nearly constant, independent of season and height of latitude.

The maximum electron density in the F layer during nighttime is then calculated by the following equation,

$$\alpha_F t' = \frac{1}{N_{F\max}} - \frac{1}{\mathfrak{N}_F'} \tag{13}$$

in which t' denotes the time after the virtual time of the cessation of the ionization.

The consideration stated above cannot strictly be applied to a case when the latitude at which the transmission takes place is very high. In such a case, some corrections deduced from experimental results are required.

(*b*) *Electron densities in the E layer*

As would be expected from the theory that the coefficient of recombination is approximately proportional to gas pressure, the value is much greater in the E layer than in the F layer. Chapman[4] has shown that the electron density in the case of the E layer is approximately given by,

$$N_E = \sqrt{\frac{I_E}{\alpha_E}} \tag{14}$$

where the suffix E denotes that these quantities are confined to the E layer.

Equation (14) represents the following fact: in the E layer the action of the recombination is so prominent and we may safely understand that the change of χ is directly accompanied with a variation of N_E without any noticeable time lag.

During nighttime N_E decreases similarly as in the case of the F layer; but, since the action of recombination is much stronger, the decrease is very prompt. The attenuation in the E layer, therefore, may be regarded to have very little effect on the high-frequency night transmission.

IV. Experimental Determination of the Coefficient of Recombination in the F Layer

It has been shown by Eckersley,[7] and several investigators[6] of the United States that the height of the F layer is roughly estimated at somewhere between 200 and 400 kilometers. Substituting, for example $Z = 300$ kilometers and $i_0 = 90$ degrees into (1), we have

$$N_0 = N_{F\max} = 1.163 \times 10^3 \cdot f_0^2 \tag{15}$$

where f_0 is to be expressed in megacycles. The equation shown above represents an important relation between the maximum electron density in the F layer and the limiting frequency of a wave.

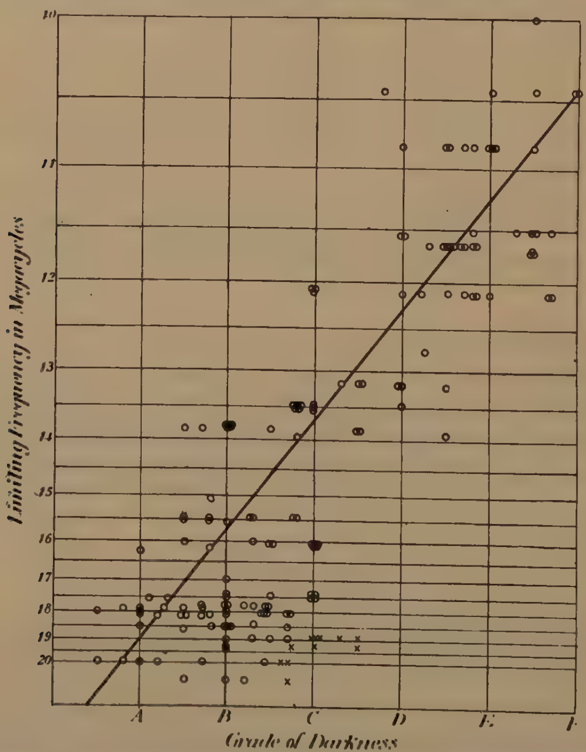

Fig. 4—Experimentally determined relation between limiting frequencies and grade of darkness.

During nighttime $N_{F\max}$ decreases with the time after the cessation of ionization. From the results of extensive observations on high-frequency field strengths[2] carried out during the last two years, the authors have obtained a very important relation of the limiting frequency to the grade of darkness, the latter being directly proportional to t',

the time measured from the instant of the virtual cessation of the ionization. In Fig. 4 the grades of darkness A, B, C, $\cdots$ etc., the full explanations of which will be given later, are taken as abscissa and the limiting frequency as ordinate, respectively. Then, calculation can be made of α_F and $\mathfrak{N}_F'$ from the curve shown in Fig. 4, by applying the relations expressed in (13) and (15). The results are given by,

$$\alpha_F = 1.5 \times 10^{-10} [\text{sec}^{-1}] \tag{16}$$

$$\mathfrak{N}_F' = 1.3 \times 10^{6} \, [\text{cm}^{-3}]. \tag{17}$$

The value of α_F thus obtained coincides fairly well with the theoretical value derived from Milne's theory, and also with the observed value given by Eckersley.[9]

It must be mentioned that, as shown in Fig. 3, the true value of the electron density at the true time of the cessation of ionization is less than $\mathfrak{N}_F'$ obtained above.

V. Calculation of Attenuation

It has already been given in Chapter II that, in the transmission of high-frequency waves, attentuation is mainly caused in the E layer and that the type of the attenuation is of *the first kind* as given in (2). A more detailed calculation of (2) will be given below, in which the result obtained in Chapter III has been taken into consideration. From (4) and (14), we have

$$N_E = N_{0E\max} \cdot e^{1/2[1-\zeta-e^{-\zeta} \cdot f(x)]} \tag{18}$$

where,

$$N_{0E\max} = \sqrt{\frac{I_{0E}}{\alpha_E}} \, .$$

Substituting (18) into (2), we have

$$\Gamma_1 = \frac{2\pi e^2}{mc\omega^2} \cdot \frac{N_{0E\max} \cdot H \cdot v}{\sqrt{\cos^2 i_0 + \dfrac{2Z_E}{r_0}}} \cdot \int_{-\infty}^{+\infty} e^{1/2[1-\zeta-e^{-\zeta} \cdot f(x)]} \cdot d\zeta . \tag{19}$$

Putting,

$$y = e^{-\zeta/2}$$

then (19) becomes,

$$\Gamma_1 = \frac{2\pi e^2}{mc\omega^2} \cdot \frac{N_{0E\max} \cdot H \cdot v}{\sqrt{\cos^2 i_0 + \dfrac{2Z_E}{r_0}}} \cdot 2\sqrt{e} \cdot \int_0^{\infty} e^{-(f(x)/2)y^2} \cdot dy . \tag{20}$$

Since (20) is a probability integral, it may easily be integrated into the following form:

$$\Gamma_1 = \frac{2\pi e^2}{mc\omega^2} \cdot \frac{\sqrt{2\pi e} \cdot H \cdot v \cdot N_{0E\max}}{\sqrt{\cos^2 i_0 + \frac{2Z_E}{r_0}} \cdot \sqrt{f(\chi)}} \tag{21}$$

$$= \frac{2\pi e^2}{mc\omega^2} \cdot \frac{1}{\sqrt{\cos^2 i_0 + \frac{2Z_E}{r_0}}} \cdot \frac{4H \cdot v \cdot N_{0E\max}}{\sqrt{f(\chi)}} \tag{22}$$

because, $\sqrt{2\pi e} \fallingdotseq 4$.

As would be understood from (22), the first kind of attenuation in the E layer can be regarded as equivalent to that which might be caused when the wave passes through a homogeneous ionized layer havng a thickness of $4H$ and the electron density of $N_{0E\max}/\sqrt{f(\chi)}$. In other words, although the actual distribution of electrons in the layer may be complicated, yet in so far as the *attenuation* is concerned, the E layer might be regarded as a simple homogeneous layer as shown by the hatched part in Fig. 5.

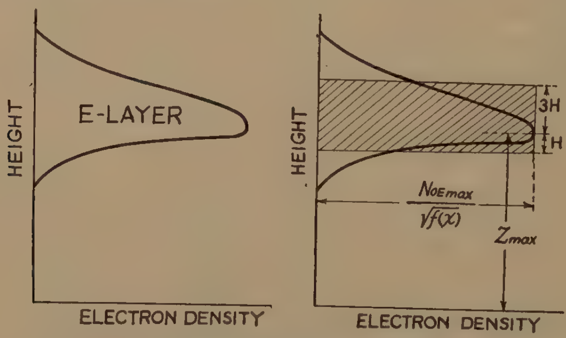

Fig. 5—Simplified distribution of free electrons assumed in calculation of attenuation.

It is shown from (22) that the attenuation is, as a general rule inversely proportional to the square of the frequency, and yet this relation is not completely realized throughout a wide range of frequencies, especially at lower frequencies. The principal reason for this may be found in the fact that in the case of the transmission of lower frequencies we have neglected not only the contributions of the energy of the waves which might be refracted back from the E layer, but also those which might be carried by higher angle rays. In the case of the transmission of waves below 10 megacycles, one part of the energy

may be refracted back from the E layer, and the arrival of a number of high-angle rays have already been experienced in the high-frequency picture transmission carried out in Germany and England. After many experimentations the authors propose a modified formula as given below for (22), which gives the closest coincidence with our empirical data.

$$\Gamma = \frac{1}{\sqrt{f(x)}}\left[\frac{k_1}{f^2} + k_2 f\right]. \tag{23}$$

Numerical values of k_1 and k_2 involved in (23) have been determined by using the result of absolute measurements shown by Burrows,[8] the final form being given as follows:

(i) For 10 megacycles $< f <$ 20 megacycles,

$$\text{Loss} = 2.9 \times 10^3 \frac{1}{\sqrt{f(x)}} \cdot \frac{1}{f^2} + 0.77 \frac{f}{\sqrt{f(x)}} \tag{24}$$

(ii) For 6 megacycles $< f <$ 10 megacycles,

$$\text{Loss} = 1.1 \times 10^3 \frac{1}{\sqrt{f(x)}} \cdot \frac{1}{f^2} + 2.6 \frac{f}{\sqrt{f(x)}} \tag{25}$$

where losses are expressed in decibels above one microvolt per meter.

VI. Ionization Chart of the Upper Atmosphere

We shall next enter into applications of the theory to practical problems. In the first place, we must prepare the *ionization chart of the upper atmosphere*; the property, use, and method of construction of the chart will be explained below.

As examples, two charts are shown in Figs. 6 and 7, the former being prepared for the *Tokio-Melbourne* circuit and the latter for the *Tokio-Honolulu-Buenos Aires* circuit. On the chart the following quantitative items are expresssed:

(i) Contour lines of the equal height of the sun during daytime.

(ii) Contour lines of the equal electron density in the F layer during nighttime.

(iii) Sunset and sunrise line.

By taking the Japan Central Standard Time as abscissa and a great-circle distance from Tokio as ordinate, respectively, the chart represents the relation between the time and the altitude of the sun during daytime, or the time and the electron density of the F layer during nighttime at any point on the great circle under consideration. In these figures lines denoted by 10, 20, 30, · · · represent the contour lines of

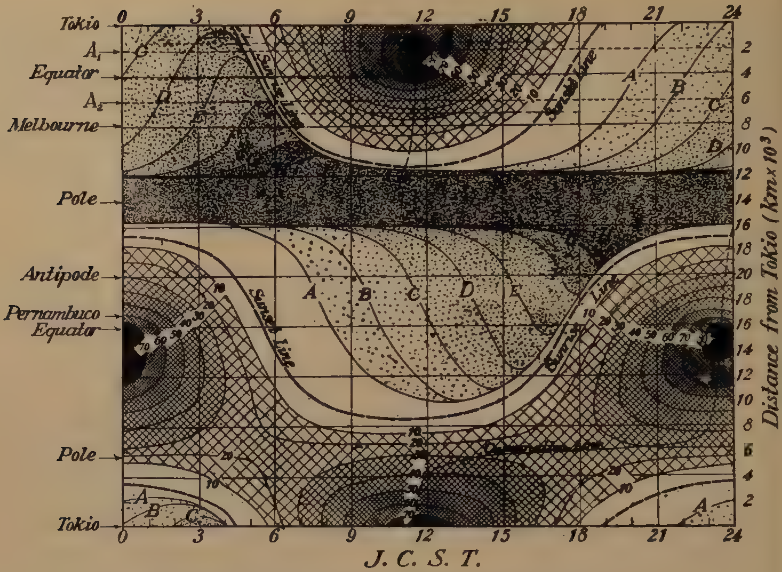

Fig. 6—Ionization chart of upper atmosphere along Tokio-Melbourne path—summer solstice.

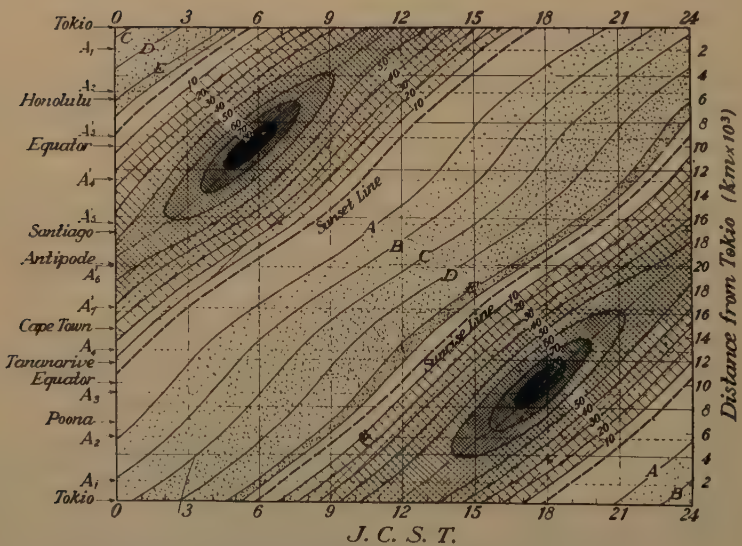

Fig. 7—Ionization chart of upper atmosphere along Tokio-Honolulu-Buenos Aires path—equinox.

equal altitude of the sun in degrees, while lines $A, B, C, D, \cdots$ denote those of equal electron density in the F layer.

Attention must be paid to the fact that the chart does not represent the dirunal change of the ionization at a specified height, but gives different quantities during day and night, respectively. In daylight it represents the change of the altitude of the sun, or what is the same as the change of the electron density in the E layer; because, as stated in Chapter III, the recombination in the E layer is so prominent that change of the zenith distance of the sun may safely be considered to be directly accompanied by the change of the electron density in the E layer. Thus it may be recognized that the chart represents the varia-

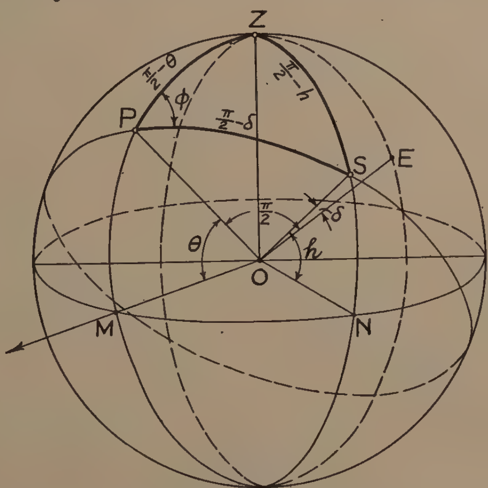

Fig. 8—Trigonometric relations used in computation of sun's altitude.

tion of N_E during daytime and the variation of N_F during nighttime, the former being a direct measure of attenuation and the latter being a principal controlling factor in night transmission. It is also to be noted that any one of the charts applies only to a specified great-circle path and a specified season.

A brief account will be given of the method of construction of the ionization chart. The method is composed of two procedures, one for the daylight part, and the other for the nocturnal part.

The great circle may first be determined when the geographical positions of transmitter and receiver are given. By using spherical trigonometry, the latitude and the longitude at any point on this great circle may be found. Then the altitude of the sun may be calculated*

* Several nomographs have been prepared by H. Iwakata, *Jour. Inst. of Teleg. and Telephone Eng. of Japan*, vol. 82, no. 1; January, (1929), giving facilities to the calculation of h.

from the following equation,

$$\sin h = \sin \delta \cdot \sin \theta + \cos \theta \cdot \cos \delta \cdot \cos \phi \qquad (26)$$

where θ, δ, and ϕ represent, respectively, the north latitude, north declination, and the local hour measured from the noon. The relations among these quantities are shown in Fig. 8, in which O represents the point under consideration, Z the zenith, S the position of the sun, and P the rotational axis of the celestial sphere.

The construction of nocturnal curves will next be taken up. Assuming $\chi_0=85$ degrees (or what is the same as $h=5$ degrees), at which the ionization is assumed to stop suddenly, a contour line of $h=5$ degrees can be drawn on the chart. By shifting this line to the right by a length representing about three to four hours, the position of A-line is settled, and again shifting A-line further to the right by two hours B-line can be determined. Similarly lines denoted by C, D, E, $\cdots$ are drawn by shifting the preceding contour line to the right by two hours, respectively.

VII. Procedure of Calculating Field Strengths

In the present chapter, a practical procedure of calculation of field strengths by the use of the ionization chart will be explained.

(1) *Determination of the direction of propagation*

As explained previously, the use of a chart is limited to only one great-circle path. Therefore, there must be prepared an infinite num-

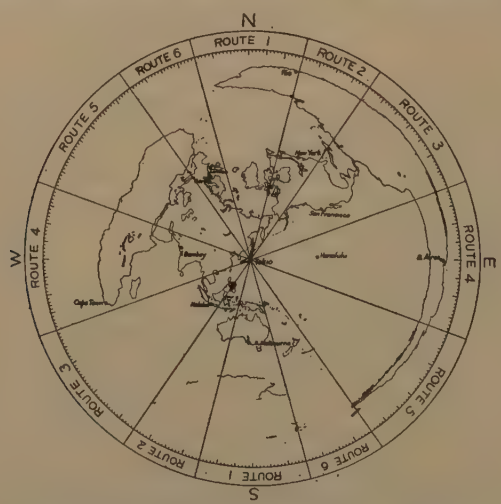

Fig. 9—Distribution of sectors selected for determination of representative ionization chart.

ber of charts in order that the projection of communication routes for all circuits between Tokio and any point on the globe may be made completely. Since it is so troublesome to prepare a large number of charts, a conventional method has been adopted by the authors. Dividing the globe into six regions as shown in Fig. 9, one chart is used for each route. For example, Route 3 in Fig. 9 involves the various transmission paths which cover the direction† of from 15 to 35 degrees or from 195 to 215 degrees, and for a communication with any point within this range one chart is used in common, although it has originally been prepared for the *Tokio-San Francisco* route. Similarly, for communications with Manila, Malabar, Mexico City, etc., the same chart may be used.

For the similar purpose of simplifying the matter with respect to the seasonal variations, the charts are prepared only for three seasons; namely winter, equinox, and summer. Eighteen charts in total are then prepared with respect to six routes and three seasons, which are printed in colors and completely shown in the full paper.[2] We may select a desired chart from one of the eighteen provided the position of the station with the communication is to be established and the season are given.

(2) *Determination of the field strength E_0 when the attenuationless transmission is assumed*

In the case of the attenuationless transmission, the field strength at the receiver may be given by the following Heltz's formula;

$$E_0 = \frac{3 \times \sqrt{W}}{d} \times 10^5 \qquad \frac{\mu V}{m} \tag{27}$$

where W and d represent the radiated power in kilowatts and a distance of transmission in kilometers, respectively; the latter may conveniently be measured along the surface of the earth without any noticeable error. The relation shown in (27) can be represented by a monograph as shown in Fig. 10, by the use of which E_0 is immediately given in decibels.

It should be mentioned here that (27) is confined to the radiation from a single dipole. When the transmitting aerial is of beam construction and composed of n elementary aerials, we must take a value of nW as the radiated power‡ in the direction of maximum radiation.

† Direction is to be measured in clockwise rotation from the true north as is commonly done in direction findings.

‡ The difference of radiation resistance of a beam aerial caused by the difference of arrangement of dipole elements is neglected in the calculation.

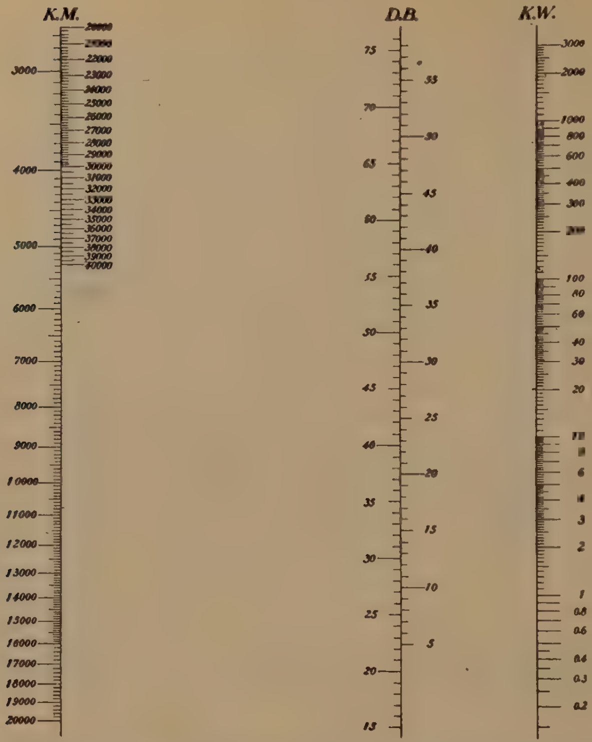

Fig. 10—Nomograph for determination of field intensity resulting from transmission through attenuationless medium.

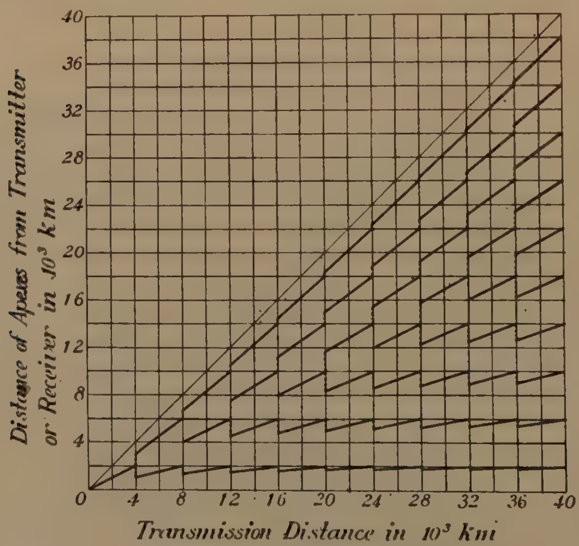

Fig. 11—Distribution of apexes along reflection path for various transmission ranges.

(3) *Determination of the positions of apexes of the ray path*

Next, the positions of the apexes of a ray path should be determined on the ionization chart; in other words, it must be known where the reflection at the F layer takes place on the transmission path. Since the wave is commonly considered to be radiated at $i_0 = 70 \sim 90$ degrees at the transmitter in the case of long-distance transmission, the distance between two consecutive apexes of the ray path extends ap-

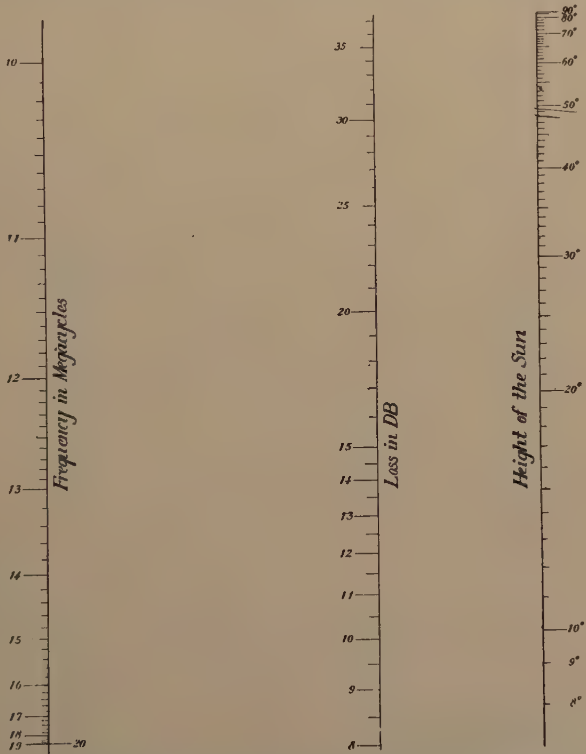

Fig. 12—Nomograph for determination of attenuation loss due to multiple reflections—high-frequency range.

proximately from 1500 to 4000 kilometers. In Fig. 11, the actual transmission distance is taken as abscissa, while the distance from the transmitter (or the receiver), at which reflection at the F layer takes place, is plotted as ordinate. For example, if the traveling distance between two stations is 13,000 kilometers, the figure shows that there occur four apexes in the complete course of the transmission, which are located at about 1700, 5200, 8600, and 12,100 kilometers, respectively, from the transmitter (or from the receiver).

(4) *Determination of the total attenuation along the ray path*

It has been shown in Chapter II that high-frequency waves pass through the E layer; and after being bent down from the F layer they again pass through the E layer before reaching the ground. It is so complicated to calculate the amounts of attenuation twice at both sides of each apex of the ray path that an assumption is conveniently made

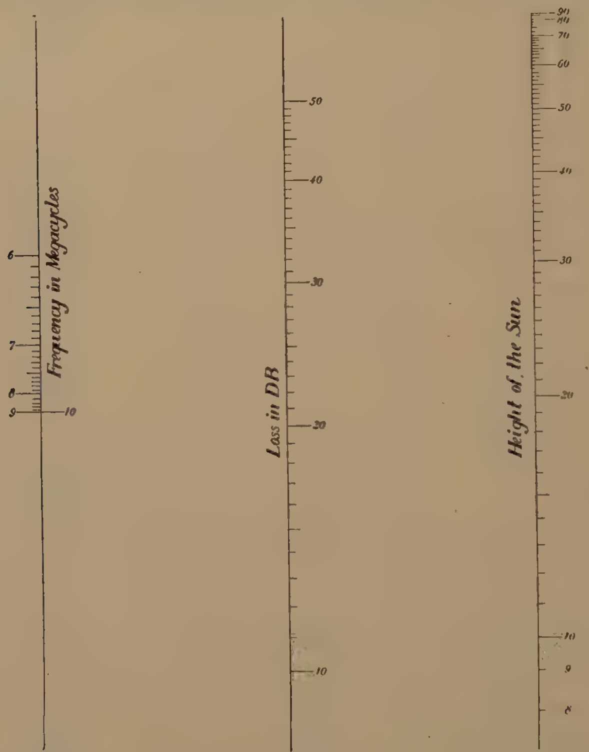

Fig. 13—Nomograph for determination of attenuation loss due to multiple reflections—low-frequency range.

by the authors; that is, the attenuation is caused only once in the complete course of one step and at the *apex* instead of the *intermediate portion* of the path.

The determination of the positions of apexes on the chart being thus made, the altitude of the sun at each apex is immediately found on the ionization chart. Then the attenuation at each apex may be calculated from (24) and (25), which are conveniently shown, in Figs. 12 and 13, as nomographs. The relations among f, h, and Γ are thus known. For example, in such a case that a wave of 12 megacycles is

eflected from the layer, at which the altitude of the sun is known to be 20 degrees, the nomograph gives a loss of 17 decibels. Similarly for a 7-megacycle wave and altitude of 20 degrees, an amount of 16.5 decibels is known to be lost.

(5) *Determination of the inaudible duration at which the field strength is nil*

The relation between the limiting frequency and the grade of darkness have been shown in Fig. 4. The curve shown in this figure may be expressed in the following formula,

$$f_0 = \frac{79}{\sqrt{0.23t + 1}} \tag{28}$$

where t is expressed in hours after the cessation of the ionization and f_0 in megacycles.

While using the chart, if the apex of the ray path is found to fall on a region at which the wave of the given frequency cannot be bent down from the F layer, the field strength at the receiver must of course be nil. For example, a 15-megacycle wave escapes outside the earth's atmosphere when the grade of darkness at the apex becomes C as shown in Fig. 4. Similarly a 19-megacycle wave escapes at the darkness of grade A.

VIII. Practical Examples of Preparing Field-Strength Curves by Using the Ionization Chart

Example 1

Preparation of a field strength curve showing a diurnal variation will next be attempted with the following data:

Transmission route . . . . . . Tokio—Cape Town
Season . . . . Spring
Frequency . . . . 10 and 15 megacycles
Radiated power . . . . 1 kilowatt
Aerial . . . . Vertical, nondirectional

Since the path under consideration belongs to Route 4 as shown in Fig. 9, and also the season of spring is chosen, the ionization chart as previously shown in Fig. 7 may be utilized.

By using the nomograph shown in Fig. 10, the field strengths in the case of the attenuationless transmission are,

$E_0 = 26$ decibels for the short path across the Indian Ocean (14,600 kilometers), and $E_0 = 22$ decibels for the long path via South America (25,400 kilometers), respectively.

(i) Transmission along the short path.

By using Fig. 11 apexes A_1, A_2, A_3, and A_4 may be marked on the chart as shown in Fig. 7.

As for the 15-megacycle wave, the escapement occurs when the darkness at the apex becomes a certain grade between B and C as shown in Fig. 4. As might be understood from the figure, the wave escapes to the outer atmosphere of the earth when any one of four apexes A_1, A_2, A_3, and A_4 falls on a region darker than the critical darkness stated above. With regards to our present case, the incom-

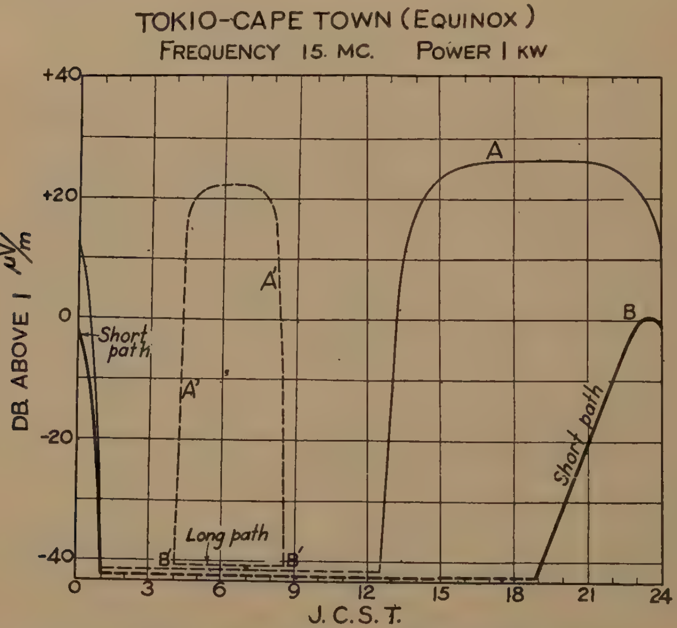

Fig. 14—Computed 15-megacycle diurnal transmission characteristics of Tokio-Cape Town path according to assumptions of Example 1.

municable duration covers nearly twelve hours of from about 0030 to 1230 J.C.S.T. The field strength level would be shown by curve A in Fig. 14, were it not for the effect of attenuation. For the reason that there arrive at the receiver a number of rays, each having a slightly different angle of incidence, the edge of the curve would be rounded off. Nevertheless, the effect of attenuation can never be neglected in practical cases; therefore it will be calculated below.

The altitude of the sun at each apex and at any time of day can immediately be taken from the chart. The values thus obtained are tabulated on the following page.

TABLE I

Value of h in degrees.

J.C.S.T. / Apex	8	10	12	14	16	18	20	22	24
A_1	10°	35°	52°	53°	38°	12°	—	—	—
A_2	—	7°	32°	57°	67°	44°	20°	—	—
A_3	—	—	8°	37°	66°	80°	46°	20°	—
A_4	—	—	—	10°	39°	68°	65°	43°	16°

The amount of attenuation at each apex and at each hour can be found from the nomograph shown in Fig. 12, the results being tabulated as follows:

TABLE II

Attenuation in decibels (frequency = 15 megacycles).

J.C.S.T. / Apex	8	10	12	14	16	18	20	22	24
A_1	10	18	22	22	19	11	—	—	—
A_2	—	8	18	22	23	20	14	—	—
A_3	—	—	9	19	23	24	20	14	—
A_4	—	—	—	10	19	23	23	20	13
Total Absorption	10	26	49	73	84	78	57	34	13

The total attenuation along the whole transmission path at each hour of a day being thus determined, the required field strength is obtained by subtracting the total attenuation loss from the attenuationless value of 26 decibels. The diurnal variation curve is shown by curve B in Fig. 4, in which the strength below -40 decibels is neglected.

Next, for a 10-megacycle wave, the escapement does not take place throughout 24 hours, because the critical darkness for the 10-megacycle wave is never met with along the whole transmission path as shown in Figs. 4 and 7. The attenuationless curve then becomes a horizontal line of 26 decibels as shown by curve A in Fig. 15.

Similar to the previous case, the amount of attenuation loss at each apex and the total loss at each hour can be tabulated as follows:

TABLE III

Attenuation in decibels (frequency = 10 megacycles).

J.C.S.T. / Ape	8	10	12	14	16	18	20	22	24
A_1	15	28	32	33	29	17	—	—	—
A_2	—	13	27	33	35	30	21	—	—
A_3	—	—	14	28	35	36	31	27	—
A_4	—	—	—	15	29	35	35	30	19
Total Absorption	15	41	74	109	128	118	87	57	19

Substracting the total loss at each hour from 26 decibels, the diurnal variation may be shown by curve B in Fig. 15.

(ii) Transmission along the long route.

Following a similar procedure as in the case of the short route, we have seven apexes A_1', A_2', $\cdots$ A_6', and A_7' as shown in Fig. 7. Without attenuation, the field strength of a 15-megacycle wave would be 22 decibels as shown by curve A' in Fig. 14, while due to the heavy attenuation along the transmission path the strength becomes far be-

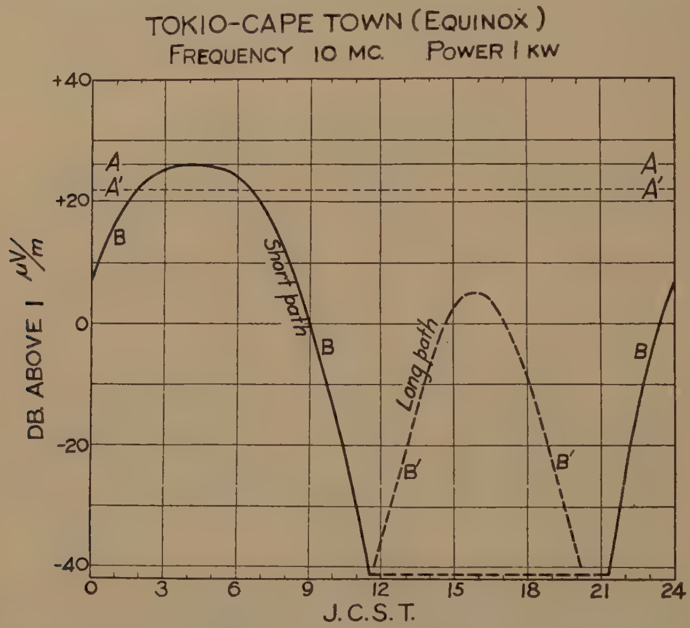

Fig. 15—Computed 10-megacycle diurnal transmission characteristics of Tokio-Cape Town path according to assumptions of Example 1.

low −40 decibels, that is, entirely nil from a practical point of view. For the 10-megacycle wave, however, the signals which have traveled along the long path can be heard as shown by curve B' in Fig. 15.

Example 2

Another example will be taken up by assuming the following conditions:

Transmission path.	Tokio-Melbourne
Season.	Summer
Frequency.	20 and 10 megacycles
Radiated power.	1 kilowatt
Aerial.	Vertical, nondirectional

Similar to the first example, the chart to be used is selected out, which has already been shown in Fig. 6. Two apexes A_1 and A_2 are marked on the chart.

For a 20-megacycle wave, the communicable duration is easily shown to be limited between about 0600 and 1930 J.C.S.T., because every one of the apexes must lie in the region brighter than grade A. Subtracting the total loss at each hour from the no-loss value of 31 decibels, the field strength curve shown in Fig. 16 is obtained. For a 10-megacycle wave, escapement does not take place as shown in the same figure.

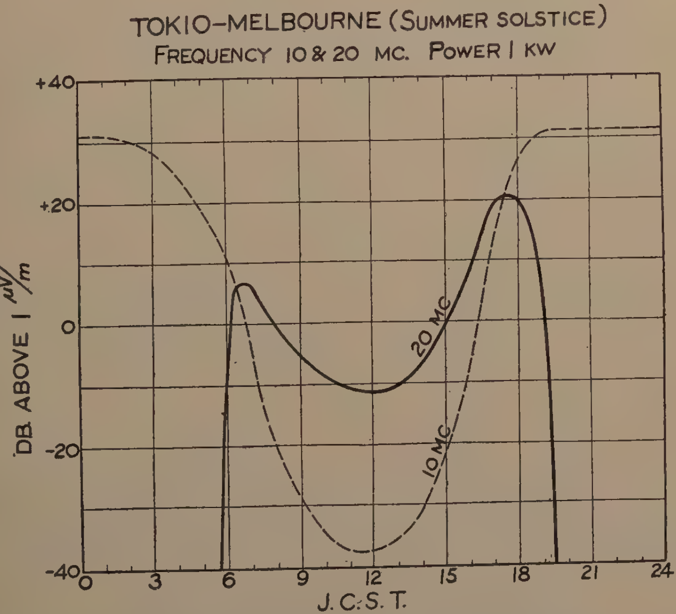

Fig. 16—Computed 10- and 20-megacycle diurnal transmission characteristics of Tokio-Melbourne path according to assumptions of Example 2.

In the summer season of the north hemisphere, the possible transmission path between Tokio and Melbourne is limited only to the short path. The ionization in the region of the south geographical pole is so meager during this season that waves of higher frequencies cannot be bent down, whereas those of lower frequencies are almost entirely absorbed when propagating in a daylight region.

Example 3

Signals from one of the transmitters at Bolinas station KLL working at $f=13.72$ megacycles were received at Tokio; and their field

strengths have been calculated by assuming that the radiated power is 20 kilowatts, the gain of the transmitting aerial 20 decibels, and the season of transmission is equinox.

The calculated result is shown in Fig. 17, which coincides fairly well with our observations.

Example 4

We shall further consider, in the case of Example 2, a problem of how the output of the transmitter and the form of the aerials must

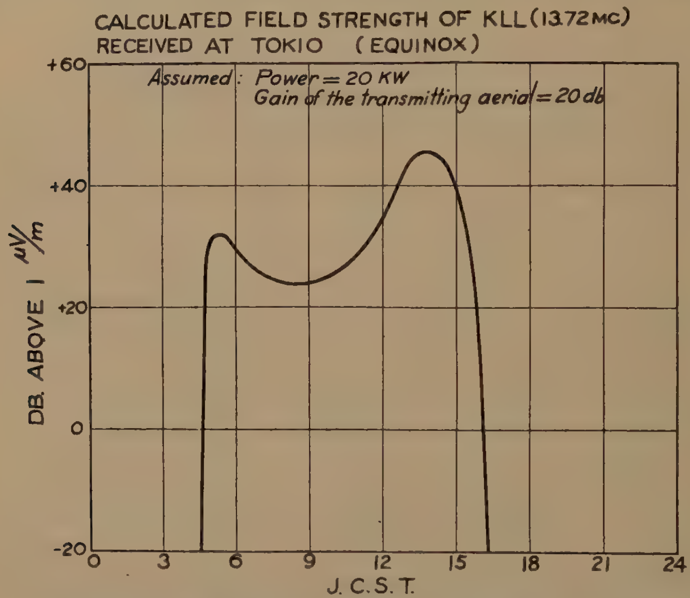

Fig. 17—Computed 13.72-megacycle diurnal transmission characteristic of Bolinas-Tokio path during the equinoctial period.

be chosen in order that the high-speed communication may be continued during 24 hours between these two links.

By high-speed communication is meant, from practical points of view, that the field strength level of about 20 decibels is always maintained at the receiving point on this frequency. Since the minimum daylight level is about -12 decibels, as shown in Fig. 16, an additional gain of $20-(-12)=32$ decibels is necessary. For trial, we distribute 12 decibels to the transmitter and 20 decibels to the sending aerial. The power of the transmitter, which has been considered to be one kilowatt, has to be raised to about 13 kilowatts in order that the gain of 12 decibels may be obtained in the field strength. Next, the transmitting aerial must be of beam construction, being composed of about 60 dipole

elements equipped with reflectors. Then the required field strength may be obtainable at the receiving point and a high-speed communication will be carried out successfully with a simple aerial for reception.

Another solution of the problem may be given as follows. The gain of 20 decibels is assumed to be divided into two halves, for each of which the transmitting and the receiving aerials are supposed to be responsible, respectively. A beam aerial, having a gain of 10 decibels is very simply constructed by arranging about 6 dipole elements fitted with reflectors. How the total gain which has to be attained for a successful communication is to be distributed between the transmitter and transmitting and receiving aerials depends upon economical consideration, and other factors such as site restriction.

At night the intensity is considerably raised due to the reduction of attenuation. But, for the reasons that on one hand the noise level is appreciably higher for the 10-megacycle wave than for a daylight wave, and on the other hand communication must always be reliable even at the transition period of changing frequencies, it may be advisable to increase further the night level of the field strength by 10 decibels, which can easily be accomplished by increasing the output of the transmitter to 10 kilowatts. In this case a dipole antenna will do for both transmission and reception.

IX. Conclusion

The theory of the transmission of high-frequency waves developed by one of the authors in the previous paper[1] is applied to practical problems, and a method is proposed for to the calculation of the field strengths of high-frequency waves, in cases where the radiated power, frequency, aerials, and season are given. Under normal conditions, in which magnetic storms are not so effective, the calculated results have been verified to be in fairly good agreement with empirical data. The proposed.method is of great use not only for a projection of a new communication service, but also for an investigation of high-frequency propagation phenomena. The complete series of ionization charts and a number of observed results are shown in the original paper.[2]

Acknowledgment

The authors wish to acknowledge their indebtedness for the able assistance of their colleagues, Messrs. S. Ueno, R. Kimura, M. Saito, J. Watanabe, T. Utsuki, and K. Nakanishi rendered in a long series of observations connected with the present study.

Bibliography

1. S. Namba, "General theory on the propagation of radio waves in the ionized layer of the upper atmosphere," *Jour. I.E.E.*, (Japan), vol. 52, p. 103; September, (1932); Proc. I.R.E., vol. 21, no. 2, pp. 238–262; February, (1933).
2. S. Namba, "Propagation characteristics of high-frequency radio waves and a method of calculating their fields strengths," published by Ministry of Communications, Electrotechnical Laboratory (Japan), Research no. 336, (1932).
3. S. Chapman, "Some phenomena of the upper atmosphere," *Proc. Roy. Soc.* (London), vol. 132, ser. A, no. A820, pp. 353–375; August 1, (1931).
4. S. Chapman, "The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth," *Proc. Phys. Soc.* (London), vol. 43, pt. 1, no. 236, pp. 26–45; January 1, (1931).
5. S. Chapman, "The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth," *Proc. Phys. Soc.* (London), vol. 43, pt. 5, no. 240, pp. 483–501; September 1, (1931).
6. T. R. Gilliland, G. W. Kenrick, and K. A. Norton, "Investigations of Kennelly-Heaviside layer heights for frequencies between 1600 and 8650 kilocycles per second," Proc. I.R.E., vol. 20, pp. 286–309; February, (1932).
7. T. L. Eckersley, "Multiple signals in short-wave transmission," Proc. I.R.E., vol. 18, pp. 106–122; January, (1930).
8. C. R. Burrows, "The propagation of short radio waves over the North Atlantic," Proc. I.R.E., vol. 19, pp. 1634–1659; September, (1931).
9. T. L. Eckersley, "Recombination of electrons and positive ions in the upper atmosphere," *Nature* (London), vol. 125, pp. 669–670; May 2, (1930).

Proceedings of the Institute of Radio Proceedings
Volume 21, Number 7 *July, 1933*

A NOTE ON NONLINEARITY IN TRANSDUCERS USED IN COMMUNICATION*

By

Peter Caporale

(Consulting Engineer, Electro-Acoustical Engineering Company of America, Philadelphia, Pa.)

Summary—*The characteristic of a nonlinear transducer can usually be expanded in a power series, when it is not simply a single term raised to some power. For those cases where the characteristic may be so expressed, a general law or equation is derived for the characteristic of a compensating transducer, that is, a transducer which in combination with the first transducer will produce over-all linearity. It is shown, moreover, that when the series for the distorting transducer is not sufficiently rapid in its convergence, it becomes impractical to realize the necessary compensating transducer.*

A simple case of an actual compensating transducer used to correct a square-law characteristic is briefly described.

Introduction

In communication systems, an ideal transducer is one whose output voltage is proportional to the input voltage and independent of frequency. Due to difficulties encountered in satisfying the frequency conditions in single elements of a transducer, combinations of elements are often used to effect some sort of "equalization." Recently attention has been directed to the possibility of similar equalization in the case of amplitude distortion. The following analysis was consequently made to determine the general relation between the characteristics of two transducers in series so as to get an over-all linear characteristic.

Theoretical Considerations

Consider the combination shown in Fig. 1. The problem may be stated thus: Suppose the functional relation between e_2 and e_1 be given,

Fig. 1

what must the corresponding relation be between e_3 and e_2 in order that e_3 be proportional to e_1; or, symbolically, if

$$e_2 = \phi_1(e_1) \tag{1}$$

$$e_3 = \phi_2(e_2) \tag{2}$$

* Decimal classification: R140. Original manuscript received by the Institute, November 21, 1932.

then, given ϕ_1, to find ϕ_2 so that

$$e_3 = ke_1. \tag{3}$$

If we express the inverse of the function[1] ϕ_1 by Ψ; i.e.

$$e_1 = \Psi(e_2) \tag{4}$$

then the condition (3) may be written:

$$e_3 = k\Psi(e_2). \tag{5}$$

The difficulty arises precisely in finding the function Ψ, which cannot always be found. Fortunately there are certain important cases in which it is possible to find Ψ from ϕ_1. An important one of these is that in which ϕ_1 is a series of the form[2]

$$e_2 = n\sum_1 a_n e_1^n. \tag{6}$$

When this series degenerates to a single term:

$$e_2 = a_1 e_1^n \tag{7}$$

we have an important condition which can, however, be solved very easily. A further simplification of (7) results in the trivial case in which

$$e_2 = a_1 e_1 \tag{8}$$

and for which ϕ_2 is obviously

$$e_3 = Be_2. \tag{9}$$

Examining (7), the inverse function Ψ is found to be

$$e_1 = \Psi(e_2) = (e_2/a_1)^{1/n}. \tag{10}$$

Substituting in (5) we get:

$$e_3 = k(e_2/a_1)^{1/n} = K(e_2)^{1/n} \tag{11}$$

where

$$K \equiv k(a_1)^{1/n}. \tag{12}$$

Thus, if transducer I has a characteristic as shown by curve A, Fig. 2(a), that is, with $n=2$, the required characteristic of transducer II, to produce over-all linearity is that shown by A' (assume $K=1$).

[1] Note that the inverse Ψ of the function ϕ_1 is a function such that $\Psi[\phi_1(e_1)] \equiv e_1$.

[2] Note that no generality is lost in writing it in this form rather than in the form $e_2 = n\sum_0 a_n e_1^n$, the difference amounting merely to a shift of coördinates.

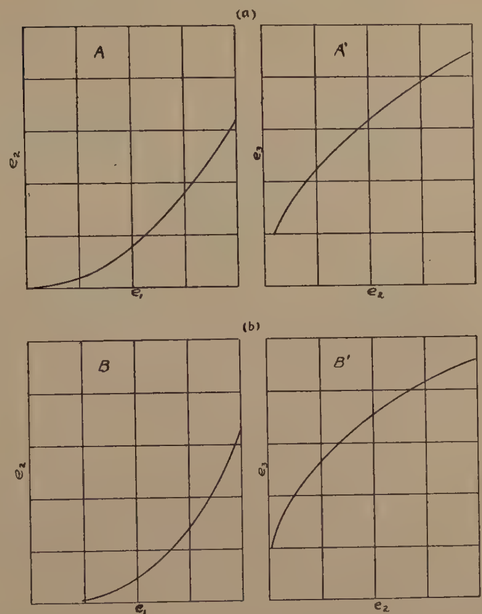

Fig. 2—Parabolic (a) and cubic (b) characteristics and their inverse.

Similarly, for $n=3$, the curves are B and B', respectively, in Fig. 2(b). It will be evident that ϕ_2 being fundamentally the inverse of ϕ_1, the operations involved in going from e_1 to e_2 will be reversed in going from e_2 to e_3. For example, if

$$e_2 = ae_1^2 \tag{13}$$

and,

$$e_1 = \alpha \cos \omega t \tag{14}$$

then,

$$e_2 = a\alpha^2 \cos^2 \omega t \tag{15}$$

$$= a\alpha^2(1 + \cos 2\omega t)/2 \tag{15a}$$

and,

$$e_3 = K\alpha\sqrt{a}\left[\frac{1 + \cos 2\omega t}{2}\right]^{1/2} \tag{17}$$

or,

$$e_3 \doteq K'\alpha \cos \omega t \tag{18}$$

$$= K'e_1 \tag{18a}$$

so that both the constant term and the double-frequency term introduced by the first transducer (see (15a)) have been eliminated, and the original frequency restored by the second transducer (see (18)).

A less obvious transformation is involved in the more general problem of (6), in which it is necessary to find the inverse function Ψ of a power series. This, however, can be done by means of Lagrange's formula, and it is found that the inverse function of a power series such as that of (6) is another power series of the form

$$e_1 = n\sum_1 b_n e_2^n \tag{19}$$

where the coefficients may be found in terms of those of (6). We shall now proceed to determine the exact nature of these coefficients.

To find the inverse function Ψ of the function[3]

$$e_2 = n\sum_1 a_n e_1^n, \tag{21}$$

let,

$$\Psi_1(e_1) \equiv n\sum_1 a_n e_1^{n-1} \tag{22}$$

so that,

$$e_2 = e_1\Psi_1(e_1) \tag{23}$$

or,

$$e_1 = e_2/\Psi_1(e_1) \equiv e_2[\chi(e_1)]. \tag{24}$$

Applying Lagrange's formula for the inverse function of a function of the form $y = x\Phi(y)$:

$$\begin{aligned} e_1 = e_2\chi(0) &+ (e_2^2/2!)\frac{d}{de_2}[\chi(e_2)]_0^2 + \cdots \\ &+ (e_2^n/n!)\frac{d^{n-1}}{de_2^{n-2}}[\chi(e_2)]_0^n + \cdots \end{aligned} \tag{25}$$

or, since $(e_2) \equiv 1/\Psi_1(e_2)$,

$$\begin{aligned} e_1 = e_2/\Psi_1(0) &+ (e_2^2/2!)\frac{d}{de_2}[1/\Psi_1(e_2)]_0^2 + \cdots \\ &+ (e_2^n/n!)\frac{d^{n-1}}{de_2^{n-1}}[1/\Psi_1 e_2)]_0^n + \cdots . \end{aligned} \tag{26}$$

$$= n\sum_1 b_n e_2^n \tag{27}$$

where,

$$b_p \equiv (1/p!)\frac{d^{p-1}}{de_2^{p-1}}[1/\Psi_1(e_2)]_0^p. \tag{28}$$

[3] See "Mathematical Analysis" by Goursat-Hedrick, vol. I, p. 406, Ginn & Co., N. Y., (1904).

To be able to evaluate b_p, let us first perform the division $1/\Psi_1(e_2)$. From (22),

$$1/\Psi_1(e_2) = \frac{1}{a_1 + a_2 e_2 + a_3 e_2^2 + \cdots} \tag{29}$$

$$= (1/a_1)\left[\frac{1}{1 + (a_2/a_1)e_2 + (a_3/a_1)e_2^2 + \cdots}\right] \tag{29a}$$

$$\equiv (1/a_1)\left[\frac{1}{1 + a_1' e_2 + a_2' e_2^2 + a_3' e_2^3 + \cdots}\right] \tag{29b}$$

where,

$$a_i' = a_{(i+1)}/a_1. \tag{29c}$$

By actual division, we can write:

$$1/\Psi_1(e_2) = (1/a_1)[1 - a_1' e_2 + (a_1'^2 - a_2')e_2^2 + \cdots] \tag{30}$$

$$\equiv {}_r\sum_0 A_r e_2^r, \tag{30a}$$

where, $A_r \equiv$ coefficients in (30). Equation (28) may thus be written:

$$b_p = (1/p!)\frac{d^{p-1}}{de_2^{p-1}}\left[{}_r\sum_0 A_r e_2^r\right]_0^p. \tag{31}$$

It is now necessary to express $[{}_r\sum A_r e_2^r]^p$ as a power series:

$$[{}_r\sum A_r e_2^r]^p = {}_m\sum c_m e_2^m \tag{32}$$

where the c's are to be determined. This may be done in the following way. From (32) we can write:

$$p \log\left[{}_r\sum_0 A_r e_2^r\right] = \log\left[{}_r\sum_0 c_r e_2^r\right] \tag{33}$$

(the replacement of m by r in the right-hand side is for convenience and can be done since they are merely summation "dummies"). Differentiating both sides of (33):

$$p\left[{}_r\sum_1 rA_r e_2^{r-1}\right]\Big/\left[{}_r\sum_0 A_r e_2^r\right]$$

$$= \left[{}_r\sum_1 mc_r e_2^{r-1}\right]\Big/\left[{}_r\sum_0 c_r e_2^r\right]. \tag{34}$$

The c's may now be found successively by equating like powers of e_2. Before proceeding to do this, we must note that in terms of these coefficients c, (31) may now be written:

$$b_p = (1/p!)\frac{d^{p-1}}{de_2^{\,p-1}}\left[\sum_{0}^{r} c_r\, e_2^{\,r}\right]_0. \tag{35}$$

Moreover, since we are only interested in the value of the derivative at the point $e_2=0$, it is evident that the only contribution to this value will be from the term in $\sum c_r e_2^{\,r}$ involving $e_2^{\,p-1}$, for the derivatives of terms of higher exponents vanish at the desired point, and the $(p-1)$th derivative of the terms of lower exponent does not exist. Hence

$$\frac{d^{p-1}}{de_2^{\,p-1}}\left[\sum_{0}^{r} c_r\, e_2^{\,r}\right]_0 = (p-1)!c_{p-1} \tag{36}$$

and,

$$b_p = (1/p!)(p-1)!c_{p-1} = c_{p-1}/p. \tag{37}$$

Returning now to (34), and clearing of fractions:

$$\begin{aligned} p[A_1 + 2A_2e_2 + 3A_3e_2^2 + \cdots][c_0 + c_1e_2 + c_2e_2^2 + \cdots] \\ = [c_1 + 2c_2e_2 + 3c_3e_2^2 + \cdots][A_0 + A_1e_2 + A_2e_2^2 + \cdots]. \end{aligned} \tag{38}$$

Equating coefficients of $e_2^{\,n}$:

$$\begin{aligned} p[A_1c_n + 2A_2c_{n-1} + \cdots + (n+1)A_{n+1}c_0] \\ = [(n+1)c_{n+1}A_0 + nc_nA_1 + \cdots + c_1A_n]. \end{aligned} \tag{39}$$

$$\text{Obviously, } c_0 = A_0^{\,p}. \tag{40}$$

Collecting terms in (39),

$$\begin{aligned} (n+1)c_{n+1}A_0 + (n-p)c_nA_1 + (n-1-2p)c_{n-1}A_2 + \cdots \\ + [n-r-p(r+1)]c_{n-r}A_{r+1} + \cdots + (1-np)c_1A_n \\ = p(n+1)A_{n+1}A_0^{\,p}. \end{aligned} \tag{41}$$

This must be true for all values of n, hence if $n=0$,

$$c_1A_0 = pA_1A_0^{\,p}$$

or,

$$c_1 = pA_1A_0^{\,p-1}. \tag{42}$$

If $n=1$,

$$2c_2A_0 + (1-p)c_1A_1 = 2pA_2A_0^{\,p}$$

and,

$$c_2 = [2pA_2A_0^p - (1-p)pA_1^2A_0^{p-1}]/2A_0$$
$$= \tfrac{1}{2}[2pA_2A_0^{p-1} + (p-1)pA_1^2A_0^{p-2}]. \quad (43)$$

For $n=2$,

$$3c_3A_0 + (2-p)c_2A_1 + (1-2p)c_1A_2 = 3pA_3A_0^p$$

and,

$$\begin{aligned} c_3 = {} & \{3pA_3A_0^p + (p-2)A_1[2pA_2A_0^{p-1} + (p-1)pA_1^2A_0^{p-2}] \\ & + (2p-1)pA_1A_2A_0^{p-1}\}/3A_0 \\ = {} & (\tfrac{1}{3})\{3pA_3A_0^{p-1} + (p-2)A_1[2pA_2A_0^{p-2} + (p-1)pA_1^2A_0^{p-3}] \\ & + (2p-1)pA_1A_2A_0^{p-2}\} \end{aligned} \quad (44)$$

etc.

Thus, given (6), we have the a's and can therefore get the A's from (30) and (30a). Having these coefficients, we can get the c's from (41), and, finally, knowing the c's we can get the coefficients b from (37). These latter are, then, the coefficients of the series which is the inverse of the series of (6).

For example, from (30) and (30a) we have:

$$\left.\begin{aligned} A_0 &\equiv 1/a_1 \\ A_1 &\equiv -(a_1'/a_1) = -(a_2/a_1) \\ A_2 &\equiv [(a_2/a_1)^2 - (a_3/a_1)]/a_1 \\ &= (a_2^2 - a_1a_3)/a_1^3 \end{aligned}\right\} \quad (45)$$

etc.

Hence from equations (40), (41), (42) and (43):

$$\left.\begin{aligned} c_0 &= A_0^p = (1/a_1)^p \\ c_1 &= -p(a_2/a_1)(1/a_1)^p \\ c_2 &= \tfrac{1}{2}[2p(a_2^2 - a_1a_3)/a_1^{p+2} + (p-1)p(a_2^2/a_1^2)(1/a_1)^{p-2}] \end{aligned}\right\} \quad (46)$$

etc.

Substituting these values in (37), we get:

$$\left.\begin{aligned} b_1 &= c_0 = 1/a_1 \\ b_2 &= \tfrac{1}{2}c_1 = -(1/a_1^2)(a_2/a_1) = -(a_2/a_1^3) \\ b_3 &= c_2/3 = (\tfrac{1}{6})[6(a_2^2 - a_1a_3)/a_1^5 + 6(a_2^2/a_1^5)] \\ &= (2a_2^2 - a_1a_3)/a_1^5 \end{aligned}\right\} \quad (47)$$

etc.

Let us now consider a vacuum tube having the following characteristic over its working range:

$$e_2 = 5e_1 + 0.001e_1^2 \tag{48}$$

where e_2 is the drop across the load in the plate circuit and e_1 is the impressed signal. This characteristic is relatively quite linear and is shown in Fig. 3 by curve A. From (47), the coefficients of the inverse series

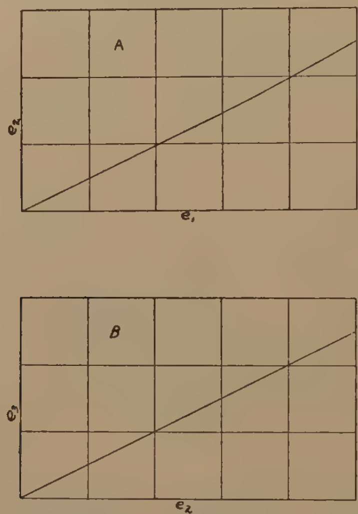

Fig. 3—Characteristic (A) of equation (48) and its inverse (B).

of (48) are:

$$\begin{aligned} b_1 &= \tfrac{1}{5} = 0.2 \\ b_2 &= -8 \times 10^{-6} \\ b_3 &= 6.4 \times 10^{-10}. \end{aligned} \tag{49}$$

This inverse series is shown by curve B in Fig. 3. If now we operate the tube over the range expressed by:

$$e_2 = 5e_1 + 0.1e_1^2, \tag{50}$$

we find that the coefficients for the inverse series are now:

$$\begin{aligned} b_1 &= 0.2 \\ b_2 &= -8 \times 10^{-4} \\ b_3 &= 6.4 \times 10^{-6}. \end{aligned} \tag{51}$$

Both (50) and its inverse are shown in curves A and B, respectively, in Fig. 4. Similarly if the curvature of the characteristic is increased so

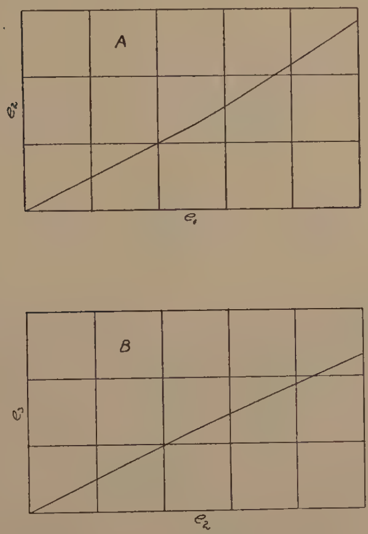

Fig. 4—Characteristic (A) of equation (50) and its inverse (B).

that

$$e_2 = 5e_1 + 0.5e_1^2 \tag{52}$$

the "b" coefficients are:

$$\begin{aligned} b_1 &= 0.2 \\ b_2 &= -4 \times 10^{-3} \\ b_3 &= 1.6 \times 10^{-4}. \end{aligned} \tag{53}$$

These two series are shown as A and B, respectively, in Fig. 5.

Note that as the curvature of the characteristic of transducer I increases, the inverse series becomes rapidly less convergent. Compare, for instance, the coefficient b_3 in (49) with the corresponding one in (53). And as the inverse series becomes less and less convergent, it becomes increasingly difficult to realize the necessary transducer in practice. Thus it is possible to correct for amplitude distortion by the introduction of the necessary compensating transducer provided the distortion is not too great; in the latter case it becomes impractical to realize a transducer having the necessary characteristic as determined above.

An interesting case of a compensating transducer for correcting amplitude distortion is pointed out in a paper by F. Massa,[4] and con-

[4] F. Massa, "Permissible amplitude distortion," to be published in the PROCEEDINGS.

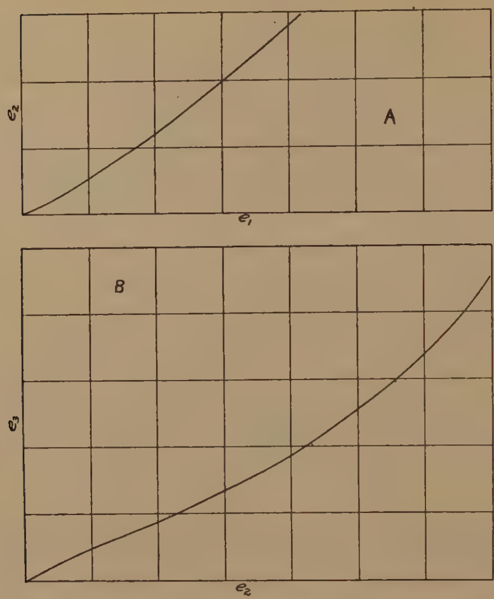

Fig. 5—Characteristic (A) of equation (52) and its inverse (B).

sists of the introduction in the plate circuit of the distorting tube an impedance which decreases with current; i.e.

$$R = kI^{-a}.$$

Thyrite, a material developed by the General Electric Company for lightning arrestor purposes, can be made to have this characteristic and the value of the exponent "a" can also be controlled in the manufacture. If, then, the characteristic of the vacuum tube is

$$I = Ae_1^2,$$

the voltage across the load R in the plate circuit will be

$$RI = RAe_1^2,$$

and if the R is due to thyrite for which the exponent "a" is one half, then,

$$RI = kA^{1/2}e_1$$

and linearity is restored. Obviously this mode of correction will not serve where the characteristic of the distorting transducer is more complicated. In any case, however, whatever the distortion may be, provided the characteristic of transducer I is a power series, the necessary characteristic of transducer II to produce over-all linearity may be found from the equations developed above.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

TABLES FOR THE CALCULATION OF THE MUTUAL INDUCTANCE OF ANY TWO COAXIAL SINGLE-LAYER COILS*

By

Frederick W. Grover
(Union College, Schenectady, N. Y.)

Summary—*Absolute formulas for the calculation of the mutual inductance of coaxial solenoids, although they are required for work of the great precision necessary in absolute measurements, are not convenient for routine calculations. Series formulas, although simpler, are limited in the range of their convergence, thus requiring the choice of the proper formula for any given problem.*

The tables and formulas here presented enable the mutual conductance of any two coaxial solenoids whatever to be calculated from a single formula. Examples make clear that a five-figure accuracy may be attained with concentric coils and even with poorly coupled coils the error does not exceed a few parts in ten thousand.

Coaxial solenoids have been much used for standards of mutual inductance and for coupled circuits in general, and considerable attention has been devoted to the calculation of their mutual inductance.[1] Absolute formulas for the general case have been derived by Nagaoka,[2] Olshausen,[3] and Terezawa,[4] and for the concentric case by Kirchhoff[3] and by Cohen.[5] These formulas involve elliptic integrals of all three kinds, and, even when arithmetico-geometrical series are employed for their evaluation, the computations are time-consuming and exacting. Thus, although the use of the absolute formulas is required in connection with absolute measurements of the highest precision, for cases where a moderate accuracy will suffice simpler methods are desirable.

For this reason series formulas have been developed by a number

* Decimal classification: R230. Original manuscript received by the Institute, January 4, 1933.

[1] A résumé of formulas for the mutual inductance of coaxial solenoids is given by E. B. Rosa and F. W. Grover, in "Formulas and tables for the calculation of mutual and self-inductance," Bureau of Standards Scientific Paper 169, Section 3, (1912). Further references to this paper will be designated as B.S. 169.

[2] H. Nagaoka, *Jour. Coll. Sci.* (Tokyo), vol. 27, art. 6, (1909).

[3] G. R. Olshausen, "Note on absolute formulae for the mutual inductance of coaxial solenoids," *Phys. Rev.*, vol. 35, p. 150, (1912). Among other formulas the author gives the correct form for a formula originally found by Kirchhoff.

[4] K. Terazawa, *Tokyo Math. Jour.*, vol. 10, p. 73; August, (1916).

[5] L. Cohen, *Bull. Bureau of Standards*, vol. 3, p. 301, (1907).

of authors,[6] especially for concentric coaxial coils. For coils coaxial, but not concentric, the formulas of Gray[7] and Clem[8] are available.

However, no single series formula is sufficiently convergent to cover all possible pairs of coils, and on account of the number of parameters involved, it is not always easy to make a choice among available formulas. In still other cases no suitable series formula has been found, and recourse must be had to the elliptic integral formulas.

The present paper has for its purpose the presentation of tables which, used in conjunction with a single formula will give the mutual inductance in any desired case with an accuracy sufficient for all purposes except those of refined absolute measurements.

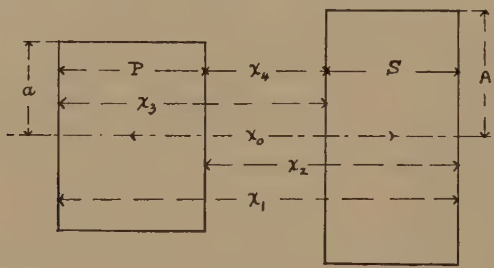

Fig. 1

The formulas here apply strictly to steady current values of the mutual inductance. For high frequencies self-capacitances of the windings will produce changes as in inductance coils in general. Since, however, single-layer coils are here assumed these disturbances will be smaller than for multiple-layer coils. For a discussion of high-frequency effects on mutual inductance standards the reader is referred to papers by Butterworth and Hartshorn.[9]

Description of the Simple Formula and the Tables

The absolute elliptic integral formulas give the mutual inductance of coaxial solenoids as a combination of four integrations, each of which

[6] Among series formulas for the mutual inductance of concentric coaxial solenoids may be mentioned the following: Maxwell, "Electricity and Magnetism," vol. II, Section 678; Roiti, see *Bull. Bureau of Standards*, vol. 3, pp. 300–310, (1907); Searle and Airey, *Electrician* (London), vol. 56, p. 318, (1905); A. Russell, *Phil. Mag.*, p. 420, April, (1907).

[7] A. Gray, "Absolute Measurements," vol. 2, part 1, p. 274, equation 53.

[8] J. Clem, "Mechanical forces in transformers," *Jour. A.I.E.E.*, vol. 46, p. 814; August, (1927).

[9] S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, vol. 33, p. 312, (1921); L. Hartshorn, "The properties of mutual inductance standards at telephonic frequencies," *Proc. Phys. Soc.*, vol. 38, p. 302, (1926).

depends upon one of the four axial distances between the end planes of one of the coils and those of the other. This suggests writing the formula for routine calculations in the form used by Clem.[8] Assume that the coils have axial lengths P and S, (see Fig. 1) radii a and A, (A being the larger), winding densities n_1 and n_2 turns per centimeter, and that the distance between their centers is x_0.

Then the distances between their ends

$$x_1 = x_0 + \frac{P+S}{2} \qquad x_3 = x_0 - \left(\frac{P-S}{2}\right)$$

$$x_2 = x_0 + \frac{P-S}{2} \qquad x_4 = x_0 - \left(\frac{P+S}{2}\right)$$

are to be calculated, and the four diagonals $r_n = \sqrt{x_n^2 + A^2}$, corresponding to the four values of x_n.

Following Clem the mutual inductance is given by

$$M = \frac{2\pi^2 a^2 n_1 n_2}{10^9}[r_1B_1 - r_2B_2 - r_3B_3 + r_4B_4] \text{ henrys} \tag{1}$$

in which the quantities B_n are to be obtained from the tables. The dimensions are in centimeters.

In the case of concentric coils $x_0 = 0$, and, since only the numerical values of the x_n are required, $x_4 = x_1 = (P+S)/2$ and $x_3 = x_2 = |(P-S)/2|$, so that for this special case,

$$M = \frac{4\pi^2 a^2 n_1 n_2}{10^9}[r_1B_1 - r_2B_2]. \tag{2}$$

The quantities B_n are functions of the radii and the spacing of the coils, and require for their specification two parameters only. (The known series formulas for the mutual inductance of *concentric* solenoids involve three parameters, and thus do not lend themselves to tabulation.) For the tables which follow the parameters chosen are $\alpha = a/A$ and $\rho_n^2 = A^2/r_n^2$, each of which must lie always between the limits zero and unity. In any given problem the value of α is the same for all the B_n, but there is a value of ρ_n^2 different for each of the values of x_n, (four for the general case and two for the concentric).

Clem[8] has given a series formula for B_n which was used in computing the values of B_n in Table I, up to the limits of its rapid convergence, but for values of α and ρ_n^2 both as great as 0.7, and larger, recourse had to be made to the absolute elliptic integral formulas.

Table I gives the values of B_n to five figures in steps of 0.05 in the

TABLE I

Values of B_n as a function of α and β^2

α	$\rho^2=1.00$	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
1.00	0.84883	0.87727	0.89552	0.91020	0.92264	0.93345	0.942975	0.95144	0.95900	0.96576	0.97180
0.95	.86783	.889825	.90561	.91859	.92971	.939435	.94805	.95573	.96261	.96877	.97428
0.90	.88418	.90175	.91531	.926665	.93655	.945245	.95298	.95990	.96612	.97169	.97668
0.85	.89870	.91296	.924555	.93444	.94314	.95085	.95774	.96393	.96951	.974515	.97901
0.80	.91176	.923445	.933285	.94185	.94944	.95622	.96231	.96781	.972765	.97723	.98124
0.75	.923565	.93318	.94150	.94885	.95542	.96132	.96668	.97151	.97588	.97983	.98338
0.70	.93426	.94217	.94917	.95543	.96107	.96618	.97082	.97503	.97884	.98230	.98541
0.65	.94394	.95045	.95629	.96157	.96637	.97074	.97472	.97835	.981645	.984635	.98732
0.60	.95270	.95803	.96286	.96727	.97130	.97499	.97837	.98146	.98427	.98683	.98913
0.55	.96060	.96492	.96888	.972515	.97586	.978935	.98176	.98435	.98672	.98887	.99082
0.50	.96769	.97115	.97434	.97730	.98003	.982555	.98488	.98702	.98897	.99076	.99237
0.45	.97400	.97673	.97927	.98163	.98382	.985845	.98772	.98945	.99103	.99248	.99379
0.40	.97958	.98169	.98366	.985495	.98721	.98880	.990275	.99164	.99289	.994035	.99508
0.35	.98444	.98603	.98751	.98890	.99020	.991415	.99254	.99358	.99454	.99542	.99622
0.30	.98862	.989765	.99084	.99186	.99280	.99369	.99451	.99527	.99598	.99662	.99721
0.25	.992125	.99291	.99365	.99435	.99500	.99561	.99618	.99671	.99720	.99765	.99806
0.20	.994975	.99547	.99594	.99638	.99680	.99719	.99755	.99789	.99821	.99849	.99875
0.15	.99718	.99746	.99772	.99797	.99820	.99842	.99862	.99881	.99899	.99915	.99930
0.10	.99875	.99887	.99899	.99910	.99920	.99930	.99939	.99947	.99955	.99962	.99969
0.05	0.99969	0.99972	0.99975	0.99977	.99980	.99982	.99985	.99987	.99989	.99995	.99992
0	1.000	1	1	1	1	1	1	1	1	1	1

α	$\rho^2=0.50$	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10	0.05	0
1.00	0.97180	0.97718	0.98194	0.98612	0.98974	0.992815	0.99535	0.99735	0.99880	0.999695	1
0.95	.97428	.97919	.98354	.98736	.99066	.99346	.99577	.99759	.99891	.99972	1
0.90	.97668	.98114	.98509	.98855	.99155	.99409	.99618	.99783	.99902	.99975	1
0.85	.97901	.98302	.98658	.98970	.99240	.99469	.99657	.99805	.99912	.99978	1
0.80	.98124	.98483	.98801	.99080	.99322	.99526	.99695	.99827	.99922	.99980	1
0.75	.98338	.986555	.989375	.99185	.99399	.99581	.99730	.99847	.99931	.99983	1
0.70	.98541	.98820	.99068	.99285	.99473	.99633	.99764	.99866	.99940	.99985	1
0.65	.98732	.98975	.99190	.993795	.99543	.996815	.99795	.99884	.99948	.99987	1
0.60	.98913	.99121	.99306	.99467	.99608	.99727	.99825	.99901	.99956	.99989	1
0.55	.99082	.99257	.99413	.995505	.99669	.99770	.99852	.999165	.99963	.999905	1
0.50	.99237	.99383	.99512	.996265	.99725	.99809	.99877	.99931	.99969	.99992	1
0.45	.99379	.99498	.99603	.99696	.99776	.998445	.99900	.99944	.99975	.99994	1
0.40	.99508	.99601	.99685	.99759	.99823	.99877	.99921	.99955	.99980	.99995	1
0.35	.99622	.99694	.99758	.99815	.99864	.99905	.99939	.99966	.99985	.99996	1
0.30	.99721	.99774	.99822	.998635	.99900	.99930	.99955	.99975	.99989	.99997	1
0.25	.99806	.99843	.99876	.99905	.99930	.999515	.99969	.999825	.99992	.99998	1
0.20	.99875	.99899	.99920	.99939	.99955	.99969	.99980	.99989	.99995	.99999	1
0.15	.99930	.99943	.99955	.99966	.99975	.999825	.99989	.99994	.99997	0.99999	1
0.10	.99969	.99975	.99980	.99985	.99989	.99992	.99995	.99997	0.99999	1.00000	1
0.05	0.99992	.99994	.99995	.99996	.99997	.99998	.99999	.99999	1.00000	1.00000	1
0	1	1	1	1	1	1	1	1	1	1	1

TABLE II

Auxiliary table for larger values of α and ρ^2

α	$\rho^2=1.00$	0.99	0.98	0.97	0.96	0.95	0.94	0.93	0.92	0.91	0.90
1.00	0.84883	0.85698	0.86298	0.86820	0.87292	0.87727	0.88133	0.88515	0.88877	0.892225	0.89552
0.99	.85294	.86035	.86606	.87107	.87562	.879825	.88376	.88747	.89100	.89436	.89757
0.98	.85686	.863665	.86910	.87391	.87829	.88236	.88617	.88978	.893205	.89647	.89960
0.97	.86063	.86693	.87210	.876715	.88094	.88487	.88857	.89207	.89539	.898575	.90162
0.96	.86428	.87014	.875065	.87949	.88356	.88736	.89094	.89433	.89757	.90066	.903625
0.95	.86783	.87329	.877985	.88223	.88615	.889825	.89329	.89658	.89972	.90273	.90561
0.94	.87127	.87639	.88086	.88494	.88872	.89226	.89562	.89881	.90186	.90478	.90759
0.93	.87462	.879435	.88370	.88761	.89125	.894675	.89792	.901015	.90397	.90681	.90954
0.92	.87788	.88242	.88649	.890245	.89375	.89706	.90020	.90320	.90607	.90883	.91148
0.91	.88107	.88536	.889245	.89285	.89622	.89942	.90246	.90536	.90815	.91082	.91340
0.90	0.88418	0.88824	0.89195	0.89541	0.89866	0.90175	0.90469	0.907505	0.91020	0.91280	0.91531

values of α and ρ_n^2. From this interpolations may be made by including second, or at most third, differences, using the Newton interpolation formula

$$f(z) = f(z_0) + hD_1 + \frac{h(h-1)}{\underline{|2}}D_2 + \frac{h(h-1)(h-2)}{\underline{|3}}D_3 + \cdots$$

in which $f(z_0)$ is the nearest tabulated value, h the fractional part of the tabular interval by which the desired argument z is separated from z_0, and D_1, D_2, D_3 are the first, second, and third differences.

In the most general case interpolation will have to be made for both α and ρ_n^2. This will be illustrated in Example 3 below.

Table II covers the range of values of α and ρ_n^2 from 0.9 to 1.0, where interpolation in Table I is difficult, with the smaller tabular interval of 0.01. In obtaining these values Dwight and Sayles' formula[10] for the mutual inductance of short concentric solenoids of equal length and nearly equal radii was used in combination with the elliptical integral formulas.

Table III has been prepared for calculations with coaxial coils of equal radii, $\alpha = 1$, with a tabular interval in ρ_n^2 of 0.01. The values in this table were calculated from elliptic integral formulas.

TABLE III

Values of B_n for coils of equal radii, ($\alpha = 1$)

ρ^2	B_n	ρ^2	B_n	ρ^2	B_n	ρ^2	B_n
.0	1.	0.25	0.992815	0.50	0.971802	0.75	0.933448
0.01	0.999987	.26	2244	.51	.970649	.76	.931397
.02	9950	.27	1650	.52	.969469	.77	.929294
.03	9889	.28	1035	.53	8262	.78	7135
.04	9804	.29	.990399	.54	7027	.79	4918
0.05	0.999695	0.30	0.989742	0.55	0.965763	0.80	0.922639
.06	9562	.31	9062	.56	4471	.81	.920297
.07	9407	.32	8360	.57	3149	.82	.917886
.08	9228	.33	7637	.58	1798	.83	5403
.09	9026	.34	6891	.59	.960416	.84	2843
0.10	0.998802	0.35	0.986123	0.60	0.959002	0.85	0.910202
.11	8556	.36	5332	.61	7558	.86	.907472
.12	8287	.37	4520	.62	6080	.87	4648
.13	7996	.38	3684	.63	4570	.88	.901721
.14	7684	.39	2826	.64	3024	.89	.898683
0.15	0.997349	0.40	0.981944	0.65	0.951443	0.90	0.895522
.16	6992	.41	1039	.66	.949826	.91	.892225
.17	6614	.42	.980110	.67	8172	.92	.888774
.18	6214	.43	.979158	.68	6480	.93	5151
.19	5793	.44	8182	.69	4748	.94	.881327
0.20	0.995351	0.45	0.977181	0.70	0.942975	0.95	0.877266
.21	4886	.46	6156	.71	.941161	.96	.872917
.22	4401	.47	5106	.72	.939302	.97	.868201
.23	3894	.48	4031	.73	7398	.98	.862983
.24	3366	.49	2930	.74	5448	0.99	.856980
0.25	0.992815	0.50	0.971802	0.75	0.933448	1.00	0.848826

[10] Dwight and Sayles, "Mutual Inductance of short concentric solenoids," *Jour. Math. and Phys.* vol. 9, no. 3, (1930).

Example 1.—Concentric coils, short secondary on the outside of a long primary, a form common in the laboratory. Given

$$a = 3 \qquad P = 50 \qquad n_1 = 10 \qquad \alpha = \frac{3}{4}$$

$$A = 4 \qquad S = 4 \qquad n_2 = 50$$

Thus,

$$x_1 = 27 \qquad x_2 = 23$$

$$r_1^2 = 745 \qquad r_2^2 = 545$$

$$\rho_1^2 = \frac{16}{745} \qquad \rho_2^2 = \frac{16}{545}$$

$$= 0.02148 \qquad = 0.02936.$$

Interpolating from Table I with $\alpha = 0.75$ and $h = 0.4296$ and 0.5872, respectively, and including second differences,

$$
\begin{array}{rlrl}
B_1 = & 0.99997 & B_2 = & 0.99994 \\
\log B_1 = & \bar{1}.9999870 & \log B_2 = & \bar{1}.9999739 \\
\log r_1 = & \underline{1.4360781} & \log r_2 = & \underline{1.3681982} \\
 & 1.4360651 & & 1.3681721 \\
\therefore\ r_1B_1 = & 27.2938_6 & a^2n_1n_2 = & 4500 \\
r_2B_2 = & \underline{23.3438_2} & & \\
\text{diff.} = & 3.9500 & & \\
\log \text{diff.} = & 0.5965971 & & \\
\log a^2n_1n_2 = & 3.6532125 & & \\
\log 4\pi^2 = & \underline{1.5963597} & & \\
\log M = & 5.8461693 & & \\
\therefore\ M = & 0.00070173 \text{ henrys} & & \\
= & 0.70173 \text{ mh.} & &
\end{array}
$$

This value has a five-figure accuracy as checked by the most precise formulas.

Example 2.—As another concentric case there may be taken

$a = 2$	$S = 24$	$n_1 = 10$	$\alpha = 0.4$
$A = 5$	$P = 30$	$n_2 = 40$	
$x_1 = 27$		$x_2 = 3$	
$r_1^2 = 754$		$r_2^2 = 34$	

$$\rho_1^2 = \frac{25}{754} = 0.03316 \qquad \rho_2^2 = \frac{25}{34} = 0.73529$$

From Table I there are found $B_1=0.99998$ and $B_2=0.98924$. Secom differences are almost inappreciable in this case. There follows

$$M = \frac{4\pi^2}{10^9}(4)(400)\left[27.4585 - 5.7682\right] = 1.37008 \text{ mh.}$$

This value is checked by the accurate series formulas,[11] which, however, have to be carried out in this case to include eight terms.

Example 3.—The following case[12] illustrates the double interpolation necessary in general.

$a = 4.435$	$S = 27.38$	$n_1 = 0.7296$	$x_0 = 31.165$
$A = 6.44$	$P = 20.55$	$n_2 = 2.737$	$\alpha = 0.6886$
$x_1 = 55.13$	$x_2 = 34.58$	$x_3 = 27.75$	$x_4 = 7.2$
$r_1^2 = 3080.79$	$r_2^2 = 1237.25$	$r_3^2 = 811.54$	$r_4^2 = 93.314$
$\rho_1^2 = 0.013462$	$\rho_2^2 = 0.033521$	$\rho_3^2 = 0.051105$	$\rho_4^2 = 0.4444$

There follow details of the interpolation for $\alpha=0.68866$ an $\rho_4^2=0.44445$. From Table I is taken the data

α	$\rho^2 = 0.40$	D_1	D_2
0.65	0.99190		
		-122	
.70	.99068		-8.5
		-130.5	
.75	$.98937_5$		

α	$\rho^2 = 0.45$	D_1	D_2
0.65	0.98975		
		-155	
.70	.98820		-9.5
		-164.5	
.75	$.98655_5$		

α	$\rho^2 = 0.50$	D_1	D_2
0.65	0.98732		
		-191	
.70	.98541		-12
		-203	
.75	.98338		

[11] B. S. 169, pp. 83–84.
[12] B. S. 169, pp. 87–90.

ince for α the value of h is 0.7732, we find B for $\rho^2=0.40$ and $=0.68866$ to be

$$.99190 + 0.7732(-\ .00122) + (-\ .0875)(-\ .000085) = 0.990965.$$

imilar calculations with $\rho^2=0.45$ and $\rho^2=0.50$ yield

ρ^2	B	D_1	D_2	
0.40	0.990965			
		-240.5		
.45	.98856		-30.5	$h = 0.8890$
		-271		
.50	.98585			

o that,

$$B_4 = 0.990965 + 0.8890(-\ .002405) + (-\ .049)(-\ .000305)$$
$$\doteq 0.98884.$$

Tables of coefficients for second and third differences facilitate the process of interpolation.

The other values are found in like manner to be $B_1=0.99999$, $B_2=0.999935$, and $B_3=0.99985$.

$r_1B_1 = 55.5043$	$r_2B_2 = 35.1723$	
$r_4B_4 = 9.5521$	$r_3B_3 = 28.4832$	Difference is 1.4009
Sum $= 65.0564$	Sum $= 63.6555$	

and M=1.0862 microhenrys.

This example illustrates the fact that, for poorly coupled coils, the mutual inductance is given by the difference of quantities considerably larger than itself. This is true of the absolute formula, and an examination of its derivation shows that the mutual inductance of two finite coils is obtained by combining mutual inductances of semi-infinite and finite coils. Formulas obtained by integrating series expressions for the mutual inductance of coaxial circular filaments show the same characteristic. Thus it is necessary to carry out the calculations of B_n more places of figures than are required in the result.

In order to avoid errors and to be able to estimate the accuracy of the result in such a case, the following variant of the calculation may be noted. If we write $B_n=1-\gamma_n$, then (1) may be written

$$M = \frac{2\pi^2a^2n_1n_2}{10^9}\begin{bmatrix} (r_1+r_4) - (r_2+r_3) \\ -\ r_1\gamma_1 - r_4\gamma_4 + r_2\gamma_2 + r_3\gamma_3 \end{bmatrix}. \tag{3}$$

The terms $(r_1+r_4)-(r_2+r_3)$ do not depend upon the tabulated values of B_n but on the given dimensions of the problem, so that the accuracy of this part of the calculation may be made as great as is necessary by

the use of a calculating machine or straight arithmetic, or use of th binomial theorem. The remaining terms are of the nature of correctio terms, and no great precision is required in their values. Carryin through the calculation for the foregoing example we find that th sum of the correction terms cannot be certain to more than four figures and the result is

$$\begin{aligned} (r_1 + r_4) - (r_2 + r_3) &= 1.50273 \\ \text{correction terms} &= -\underline{0.10180} \\ & 1.4009 \end{aligned}$$

and the last figure may be slightly in error.

This is confirmed by a calculation by the absolute formula[12] whic gives $M = 1.0865$ microhenrys, and if the B_n be calculated directly in stead of being interpolated from the table, the value 1.0864 is found It is interesting to note that, using seven-place logarithms with th absolute formula, it is difficult to obtain an accuracy beyond the fifth place.

Example 4.—Coaxial coils of equal radii, using Table III. Such coil are often used, on account of the convenience of winding them on th same form. Given,

$$a = A = 20 \quad P = 4 \quad x_1 = 15 \quad x_3 = 9 \quad \alpha = 1$$
$$x_0 = 10 \quad S = 6 \quad x_2 = 11 \quad x_4 = 5$$
$$\rho_1^2 = 0.64 \; \rho_2^2 = 0.767754 \; \rho_3^2 = 0.831601 \; \rho_4^2 = 0.941176$$

From Table III

$$B_1 = 0.953024, \quad B_2 = 0.929772, \quad B_3 = 0.914998, \quad B_4 = 0.88086$$
$$\gamma_1 = 0.046976, \quad \gamma_2 = 0.070228, \quad \gamma_3 = 0.085002, \quad \gamma_4 = 0.11913$$

Calculating the r_n and the correction terms separately

$$\begin{aligned} (r_1 + r_4) - (r_2 + r_3) &= 0.858396 & -\gamma_1 r_1 &= -1.17440 \\ \text{correction terms} &= -\underline{.16328} & -\gamma_4 r_4 &= -\underline{2.45611} \\ \text{diff.} &= 0.69512 & \text{sum} &= -\underline{3.63051} \\ & & \gamma_2 r_2 &= 1.60300 \\ & & \gamma_3 r_3 &= \underline{1.86423} \\ & & \text{sum} &= 3.46723 \end{aligned}$$

$$\begin{aligned} M &= \frac{2\pi^2}{10^6} a^2 n_1 n_2 (0.69512) \\ &= \frac{4\pi a}{10^6} N_1 N_2 (0.90991) \mu\text{h} \end{aligned}$$

in which N_1 and N_2 are the numbers of turns on the coils.

An examination of the correction terms indicates that an accuracy of the order of one part in one hundred thousand may be expected in this result. This is confirmed by making the calculation using self-inductance formulas,[13] which is possible with coils of equal radii. The result is given as a difference of larger terms in this case also, but by carrying each term to eight figures, the mutual inductance is found to be $4\pi a/10^6(0.909926)N_1N_2$ microhenrys.

As further methods of checking the mutual inductance of poorly coupled coaxial solenoids may be cited the following. To a first approximation the mutual inductance is equal to the product of the numbers of turns by the mutual inductance of two circular coaxial filaments, one placed at the central turn of each coil. A better approximation is to replace each coil by two coaxial circular filaments, using the method of Lyle.[14] Closer results still are obtained by subdividing the coils and replacing each subdivision by two Lyle filaments. Another possibility is to replace each coil by a number of equally spaced circular filaments, and to sum the mutual inductances of these by some method of mechanical integration, such as Simpson's rule. For these methods tables of the mutual inductances of coaxial circular filaments are useful.[15]

These methods apply to coaxial coils of unequal radii, but there will be quoted here the results for the coils of equal radii just treated. The following results were found.

$$\text{Central filaments}\quad \frac{10^6M}{4\pi aN_1N_2} = 0.88539$$

$$\begin{aligned}
&\text{Two filaments each coil} && = 0.90966\\
&\text{Four filaments each coil} && = 0.909905\\
&\text{Filaments 1 cm apart} && = 0.909937\\
&\text{By self-inductance formulas} && = 0.909926\\
&\text{By Rosa's correction method} && = 0.909925\\
&\text{By Table III} && = 0.90991
\end{aligned}$$

Many further examples have been treated, but it is believed that the foregoing sufficiently illustrates the use of the tables for routine calculations and the order of accuracy which is attainable.

[13] B. S. 169, pp. 94–97.
[14] Lyle, *Phil. Mag.*, vol. 3, p. 310, (1902).
[15] Grover, Bureau of Standards Science Paper 498.

Proceedings of the Institute of Radio Engineers
Volume 21, Number 7 *July, 1933*

SOME GENERAL RESONANCE RELATIONS AND A DISCUSSION OF THEVENIN'S THEOREM*

By

J. G. Brainerd

(Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania)

Summary—*It is shown that the maximum possible current which can be obtained through an impedance joining two points of a linear network is in general greater than the short-circuit current between those points; the maximum possible voltage is in general greater than the open-circuit voltage; the ratio of open-circuit voltage to short-circuit current is equal to the ratio of the maximum possible voltage to the maximum possible current, and both are equal to the magnitude of the input impedance of the network, etc. The condition for maximum voltage across a branch of a linear network is derived.*

As a preliminary, a form of Thevenin's theorem different from the usual one and of greater usefulness in the analytical solution of some circuit problems is obtained.

THEVENIN'S theorem (sometimes called Pollard's theorem) is extremely useful in electrical circuit *theory* and of much less utility in the solution of specific network problems. In this it resembles most principles, other than Kirchhoff's laws and the superposition theorem, which relate to the distribution of steady-state currents and voltages in linear networks. One reason is that such principles —reciprocity theorem, Thevenin's theorem, compensation theorem, etc.,—do not materially reduce the labor required to solve network problems by direct application of Kirchhoff's laws, although there are many exceptions to this generalization.

To illustrate, Thevenin's theorem as usually stated is: If an impedance $\mathbf{z}_r$ be connected between any two points of a circuit, the resulting current through $\mathbf{z}_r$ is the potential difference between the two points when there is infinite impedance between them, divided by the sum of $\mathbf{z}_r$ and the impedance of the network measured between the two points.[1] Hence the current through and the voltage across an impedance $\mathbf{z}_r$ may be anticipated by measurements made at the points to which $\mathbf{z}_r$ is to be connected. When, however, the current and voltage are to be determined analytically rather than by measurement, the calculation of the input impedance usually necessitates the expenditure of as much effort as required to obtain the current and voltage

* Decimal classification: R140. Original manuscript received by the Institute, March 7, 1933.

[1] Shea, "Transmission Networks and Wave Filters," p. 55, Van Nostrand, (1929).

Complex quantities are denoted by bold-face symbols, corresponding magnitudes are light face.

directly.[2] Thus despite the fact that the open-circuit voltage may be relatively easy to obtain, there frequently is no advantage in the use of Thevenin's theorem in such a case. It is the purpose here to point out that a knowledge of open-circuit voltage *and short-circuit current* is sufficient to allow the use of the theorem, and to give some simple but interesting resonance relations which follow from its application.

Let 11′ be the input and 22′ the output terminals of a passive linear transducer (Fig. 1), and let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electromotive forces and

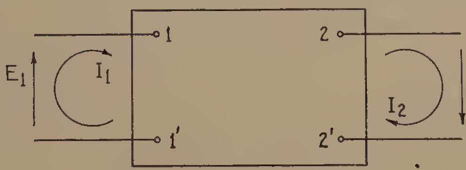

Fig. 1—Transducer.

$\mathbf{I}_1$ and $\mathbf{I}_2$ the currents indicated. Then[3]

$$\mathbf{I}_2 = \frac{\mathbf{E}_1}{\mathbf{z}_{1t}} = \frac{\mathbf{E}_1\mathbf{z}_{2s}}{\mathbf{z}_{ts}(\mathbf{z}_{2s} + \mathbf{z}_r)} \tag{1}$$

where $\mathbf{z}_{1t}$ is the transfer impedance $\mathbf{E}_1/\mathbf{I}_2$ when $\mathbf{E}_2 = -\mathbf{z}_r\mathbf{I}_2$ ($\mathbf{z}_r$ is the receiver impedance), $\mathbf{z}_{ts}$ is the (short-circuit) transfer impedance $\mathbf{E}_1/\mathbf{I}_2$ when $\mathbf{E}_2 = 0$ or $\mathbf{E}_2/\mathbf{I}_1$ when $\mathbf{E}_1 = 0$. It follows that

$$-\mathbf{E}_2 = \mathbf{z}_r\mathbf{I}_2 = \frac{\mathbf{E}_1\mathbf{z}_{2s}\mathbf{z}_r}{\mathbf{z}_{ts}(\mathbf{z}_{2s} + \mathbf{z}_r)} \tag{2}$$

whence the open-circuit voltage at 22′ is

$$-\mathbf{E}_{20} = \frac{\mathbf{E}_1\mathbf{z}_{2s}}{\mathbf{z}_{ts}}. \tag{3}$$

Hence (1) contains Thevenin's theorem.

The ratio $\mathbf{E}_1/\mathbf{z}_{ts}$ is the current $\mathbf{I}_2$ when $\mathbf{E}_2 = 0$. Writing this short-circuit current as $\mathbf{I}_{2s}$, (3) gives

$$-\mathbf{E}_{20} = \mathbf{I}_{2s}\mathbf{z}_{2s} \quad \text{or} \quad \mathbf{z}_{2s} = -\mathbf{E}_{20}/\mathbf{I}_{2s}. \tag{4}$$

The open-circuit voltage at 22′ is the product of the short-circuit current between 22′ and the input impedance at 22′; the latter is the

[2] Wenner, "A principle governing the distribution of current in systems of linear conductors," Bureau of Standards Scientific Paper No. 531, vol. 21, (1926), gives examples in which the use of Thevenin's theorem, as stated above, facilitates solving certain problems.

[3] Brainerd, "Relations between the parameters of coupled-circuit theory and transducer theory," Proc. I.R.E., vol. 21, no. 2, p. 282, February, (1933).

ratio of this open-circuit voltage to the short-circuit current.[4] Thus $\mathbf{z}_{2s}$ can be found when $\mathbf{E}_{20}$ and $\mathbf{I}_{2s}$ are known.

Now the determination of open-circuit voltages and short-circuit currents is frequently of less difficulty than the calculation of other voltages and currents or of input impedances. Therefore the simple relation expressed by (4) may represent a considerable saving of labor in solving specific problems. An example is given in the appendix. However, a knowledge of any pair of the parameters $\mathbf{z}_{2s}$, $\mathbf{I}_{2s}$, $\mathbf{E}_{20}$, is sufficient for the application of Thevenin's theorem, and in the (unusual) event that $\mathbf{z}_{2s}$ and $\mathbf{I}_{2s}$ are simpler to find than $\mathbf{E}_{20}$, the theorem may be stated

$$\frac{\mathbf{I}_2}{\mathbf{I}_{2s}} = \frac{1}{1 + \dfrac{\mathbf{z}_r}{\mathbf{z}_{2s}}}. \tag{5}$$

Likewise when $\mathbf{E}_{20}$ and $\mathbf{z}_{2s}$ are known, the theorem may be written

$$\frac{\mathbf{E}_2}{\mathbf{E}_{20}} = \frac{1}{1 + \dfrac{\mathbf{z}_{2s}}{\mathbf{z}_r}}. \tag{6}$$

Since $\mathbf{z}_{2s}$ is a property of the network, it is the ratio of the open-circuit voltage and short-circuit current at 22′ due to *any* electromotive force. Consequently to determine $\mathbf{z}_{2s}$ the actual electromotive force may be short-circuited and another assumed in any branch of the network which is convenient. It sometimes happens that this expedites the calculation of $\mathbf{z}_{2s}$.

Thevenin's theorem shows that $\mathbf{E}_{20}$, $\mathbf{I}_{2s}$, $\mathbf{z}_{2s}$, may be considered fundamental parameters which determine effects in $\mathbf{z}_r$. It is of interest to relate resonance voltage and current to these. For a given transducer with output terminals 22′, maximum power is obtained at the receiver

[4] These results can be seen from physical considerations, or can be deduced from Thevenin's theorem by taking $\mathbf{z}_r = 0$. Equation (1) has been used because it shows that there are many different expressions for $\mathbf{I}_2$ and $\mathbf{E}_2$, each with a different physical interpretation. These may be obtained by substituting for $\mathbf{z}_{ts}$ and $\mathbf{z}_{2s}$ their equivalents in terms of other transducer parameters. The usual statement and proof of Thevenin's theorem does not indicate these, hence (1) may be considered a statement which includes, and which is more general than, Thevenin's theorem. As an example of a "theorem" similar to the forms of Thevenin's theorem given in (5) and (6) below, the ratio of current ratios may be cited. Let σ_1 be the ratio of output to input current $(\mathbf{I}_2/\mathbf{I}_1)$ and let σ_{1s} be the value of σ_1 when $\mathbf{z}_r = 0$. Then

$$\frac{\sigma_1}{\sigma_{1s}} = \frac{1}{1 + \dfrac{\mathbf{z}_r}{\mathbf{z}_{20}}}$$

where $\mathbf{z}_{20}$ is the impedance at 22′ when 11′ are open-circuited.

when $\mathbf{z}_r$ is the conjugate of $\mathbf{z}_{2s}=r_{2s}+jx_{2s}$, maximum current when $\mathbf{z}_r=-jx_{2s}$, maximum voltage when $\mathbf{z}_r=-jz_{2s}^2/x_{2s}$. The maximum power and maximum current conditions are commonplace; the maximum voltage condition[5] can be obtained thus: Equation (2) indicates that E_2 would be maximum when $\mathbf{z}_r=-\mathbf{z}_{2s}$, which is not physically realizable when $\mathbf{z}_{2s}$ is not a pure imaginary. Assuming the real part of $\mathbf{z}_r$ not negative, for a given modulus z_r the numerator is unchanged in magnitude by change in phase of $\mathbf{z}_r$, the denominator has minimum magnitude when $\mathbf{z}_r$ lies as nearly as possible in the j or $-j$ direction, depending on whether jx_{2s} is negative or positive. Hence assuming $\mathbf{z}_r=jx_r$, substituting in (2) and differentiating E_2, the condition for E_2 maximum follows.

(a) *Voltage*—Substituting $\mathbf{z}_r=-jz_{2s}^2/x_{2s}=-jz_{2s}/\sin\theta_{2s}$ in (2),

$$\frac{\mathbf{E}_{2\max}}{\mathbf{E}_{20}}=\frac{1}{\cos\theta_{2s}}\underline{\big|-\theta_{2s}} \tag{7}$$

where $\mathbf{E}_{2\max}$ is the vector maximum voltage obtainable across $\mathbf{z}_r$. $E_{2\max}$ is not less than E_{20}. The relation between $\mathbf{E}_{20}$ and the input voltage $\mathbf{E}_1$ to the transducer is given by (3), and it is seen that by making z_{2s} large and z_{ts} small the ratio E_{20}/E_1 is made large, and by making θ_{2s} as near $\pi/2$ as possible (minimizing resistance component of $\mathbf{z}_{2s}$) the ratio $E_{2\max}/E_{20}$ is made large. The current through $\mathbf{z}_r$ when $\mathbf{E}_2$ is maximum is

$$(\mathbf{I}_2)_{E_{2\max}}=j\mathbf{I}_{2s}\tan\theta_{2s}. \tag{8}$$

(b) *Current*—Substitute $\mathbf{z}_r=-jx_{2s}$ in (1).

Then,

$$\frac{\mathbf{I}_{2\max}}{\mathbf{I}_{2s}}=\frac{1}{\cos\theta_{2s}}\underline{\big|\theta_{2s}}=-\frac{\mathbf{E}_{20}}{\mathbf{r}_{2s}} \tag{9}$$

where $\mathbf{I}_{2\max}$ is the vector maximum possible current through $\mathbf{z}_r$. $I_{2\max}$ is not less than I_{2s}. The voltage across $\mathbf{z}_r$ when the current is maximum is

$$(\mathbf{E}_2)_{I_{2\max}}=-j\mathbf{E}_{20}\tan\theta_{2s}. \tag{10}$$

It is interesting to note that $I_{2\max}$ (obtained when $\mathbf{z}_r=-jx_{2s}$) and $E_{2\max}$ (obtained when $\mathbf{z}_r=-jx_{2s}/\sin\theta_{2s}$) are related by

$$E_{2\max}=z_{2s}I_{2\max} \tag{11}$$

and that (10) is (8) multiplied by $\mathbf{z}_{2s}$.

[5] Apparently not discussed in the literature. In texts giving the power and current conditions, specific problems of voltage maximization are occasionally solved by differentiation. As far as the author is concerned, the condition was evolved in a class discussion. In particular, credit is due Bernard Miller, senior student in the Moore School.

(c) *Power*—When $\mathbf{z}_r = r_{2s} - jx_{2s}$,

$$I_2 = I_{2\max}/2 \quad \text{and} \quad E_2 = E_{2\max}/2. \tag{12}$$

Appendix

As an illustration[6] of the use of the short-circuit current in the solution of network problems, the current in the indicator arm of a simple bridge circuit (Fig. 2) will be found. Let $\mathbf{z}_a, \mathbf{z}_b, \cdots, \mathbf{z}_e$ be the impedances and $\mathbf{E}$ the electromotive force indicated. The open-circuit voltage and short-circuit current are by inspection

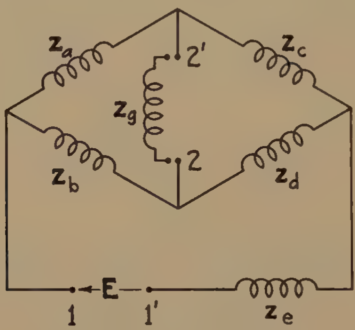

Fig. 2—Bridge circuit.

$$\mathbf{E}_{20} = \left[\frac{\mathrm{a(b+d)}}{\mathrm{a+b+c+d}} - \frac{\mathrm{b(a+c)}}{\mathrm{a+b+c+d}}\right]\mathbf{I}; \qquad \mathbf{I} = \frac{\mathbf{E}}{\dfrac{\mathrm{(a+c)(b+d)}}{\mathrm{a+b+c+d}} + \mathrm{e}}$$

$$\mathbf{I}_{2s} = \left[\frac{\mathrm{a}}{\mathrm{a+b}} - \frac{\mathrm{c}}{\mathrm{c+d}}\right]\mathbf{I}'; \qquad \mathbf{I}' = \frac{\mathbf{E}}{\dfrac{\mathrm{ab}}{\mathrm{a+b}} + \dfrac{\mathrm{cd}}{\mathrm{c+d}} + \mathrm{e}}.$$

Therefore, the current through an indicator of impedance $\mathbf{z}_g$ inserted between 2 and 2′ is, since $\mathbf{z}_{2s} = -\mathbf{E}_{20}/\mathbf{I}_{2s}$,

$$\mathbf{I}_g = \frac{-\mathbf{E}_{20}}{\mathbf{z}_{2s} + \mathbf{z}_g}$$

$$= \frac{\mathrm{(ad+bc)}\mathbf{E}}{\mathrm{(a+b)(c+d)e+ab(c+d)+cd(a+b)+g\,[(a+c)(b+d)+e(a+b+c+d)]}}$$

[6] The problem is chosen because it is frequently used to illustrate Thevenin's theorem. Wenner, *loc. cit.*, gives it, leaving $\mathbf{z}_{2s}$ in $\mathbf{I}_g$ undetermined in the general case. Below a is written for $\mathbf{z}_a$ etc.

BOOK REVIEWS

Short-Wave Wireless Communication, by A. W. Ladner and C. R. Stoner. Published by John Wiley and Sons, Inc. Price, $3.50.

The purpose of the authors is to supply a textbook to meet the needs not only of engineers and telegraphists engaged in wireless, but also of the scientific amateur and those who have already an outline knowledge of long wave working.

The first part of the book concerns itself with a brief history of short-wave development followed by a general discussion of the behavior of electromagnetic waves and the propagation of short wireless waves. It points out that the prediction of the performance of a short-wave circuit or the choice of a suitable wavelength is a difficult matter, and includes a method for obtaining the approximate field strength to be expected for different wavelengths over various wireless routes from England to other parts of the world.

The next few chapters are given over to a discussion of the modulation of high-frequency waves, the relationship of carrier to side bands, push-pull operation of valves, and self-oscillators.

Considerable space is devoted to the operation of a short-wave system covering such matters as driven circuits, constant-frequency oscillators, and modulation circuits. The proper terminations and junctions of high-frequency feeders are described in some detail, and equations for the current and voltage along the feeders are worked out. It is pointed out that concentric types of feeders are coming into more general use and a brief description of the Marconi feeder system is included. Under aerials and aerial arrays a description is given of the various types of arrays in use to-day.

There follows a brief discussion of simple receivers and receiver problems and a chapter each on commercial receivers and transmitters. Considerable space is devoted to methods of diversity reception, and such problems as proper telephone terminal equipment, privacy, and selective fading are briefly touched upon. The new Marconi transmitter having several unique features which has been installed in the League of Nations Wireless Station at Prangins is described. This set can transmit on any one of four "spot" waves, each adjusted to the required frequency precision, the time taken to change from one to the other being only two minutes. It is designed to operate on either telegraphy or telephony and can be changed from one to the other instantly.

The book closes with a chapter on ultra-short waves, giving briefly their peculiarities and outlining different ways of transmitting and receiving them.

A short appendix of mathematical calculations is added and selected references on each subject will be found at the end of each chapter.

*Isabel S. Bemis

* East Orange, N. J.

Annual Tables of Constants and Numerical Data—Chemical, Physical, Biological, and Technological. Section on Radio, Extracted from Volume VIII (1927–1928) prepared by M. Mesny. 18 pages. Price of radio excerpt, bound, 30 francs. Gauthier-Villars and Company, 55 Quai des Grands Augustins, Paris VI.

The Annual Tables of Constants and Numerical Data are compiled under the direction of an international committee. The section on radio, available as a separate bound volume, as indicated above, consists of data taken from various technical radio publications, including the PROCEEDINGS of the Institute of Radio Engineers, the *Proceedings of the Royal Society*, *Zeitschrift für Hochfrequenz*, *L'Onde électrique*, and others. The data, while not extremely comprehensive, relate to a variety of subjects, including the following: vacuum tubes, quartz resonators and magnetostriction oscillators, wave propagation and field intensity measurements, Kennelly-Heaviside ionized layer data, atmospherics, and dispersion of short waves.

L. E. WHITTEMORE

Standards Year Book, 1933, U. S. Department of Commerce, Bureau of Standards, Miscellaneous Publication No. 139. 250 pages. Price $1.00. Superintendent of Documents, Government Printing Office, Washington, D. C.

The Standards Year Book for 1933 is the Seventh Edition in this series published by the U. S. Department of Commerce. Its scope is indicated by the following outline of its contents:

Chapter	*Contents*
I.	Establishing and Maintaining Standards
II.	International Standardizing Agencies
III.	National Standardizing Laboratories
IV.	National Industrial Standardizing Bodies
V.	Federal Standardizing Agencies (U.S.A.)
VI.	Bureau of Standards
VII.	County and State Purchasing Agencies
VIII.	General Standardizing Agencies (U.S.A.)
IX.	Standardizing Activities and Technical Societies and Trade Associations
X.	Bibliography on Standardization
	Index

References to radio include a brief résumé of the radio standardization work of the Bureau of Standards. The chapter relating to the standardizing activities of technical societies and trade associations contains reference to the activities of the Radio Manufacturers Association.

†L. E. WHITTEMORE

† American Telephone and Telegraph Company, New York City.

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

Copies of the publications listed on this page may be obtained gratis by addressing the manufacturer or publisher.

Rectox battery charges are described in a bulletin issued by the Westinghouse Electric and Manufacturing Company of East Pittsburgh, Pa.

A leaflet has been issued by the General Radio Company of 30 State Street, Cambridge, Mass., covering their Type 653 volume controls.

A leaflet on relays for various purposes has been issued by Signal Engineering and Manufacturing Company, 154 W. 14th Street, New York City.

Standardized service units are described in Circular S1 published by the Weston Electrical Instrument Corporation of 614 Frelinghuysen Avenue, Newark, N. J.

Standard designs of Jensen speakers are described in a leaflet issued by the Jensen Radio Manufacturing Company of 6601 S. Laramie Avenue, Chicago, Illinois.

Sheet No. 10137 issued by the Emerson Electric Manufacturing Company of St. Louis, Mo., describes the Emerson "B" power unit for operation from 32 volts direct-current systems.

Circular No. 15 on the distribution of weather information, forecasts, and warnings by radio for the benefit of navigation on the Great Lakes may be obtained from any weather bureau office at Great Lake ports or from the Department of Agriculture, Weather Bureau, Washington, D. C.

Data sheets are available from the S. S. White Dental Manufacturing Company of 152 W. 42nd Street, New York City, on flexible shafting for remote control of radio equipment as well as the various types of resistors manufactured by them.

Plugs and recepticals of many varieties are described in a leaflet issued by the Canon Electric Development Company of 420 W. Avenue 22, Los Angeles, Calif.

A line of fixed paper dielectric condensers is announced in a booklet by Wego Company, Ltd., Perivale, Greenford, Middlesex, England.

Condensers and resistors of various types and sizes are covered in a pamphlet issued by the Aerovox Corporation of 70 Washington Street, Brooklyn, N. Y.

The Miles Reproducer Company of 244 W. 23rd Street, New York City, issues a catalog covering public address systems and various components for these.

Piezo-Electric Laboratories of 612 Rockland Avenue, New Dorp, N. Y., have issued a leaflet on their precision crystal mountings.

DeJur-Amsco Corporation, 95 Morton Street, New York City, have issued leaflets covering their adjustable and variable condensers.

The technical data booklet on tube characteristics is issued by the Arcturus Radio Tube Company of Newark, N. J.

The Acheson Oildag Company of Port Huron, Michigan, has issued its Bulletin No. 102 G covering the lubrication of small mechanical devices with colloidal graphite.

RADIO ABSTRACTS AND REFERENCES

THIS is prepared monthly by the Bureau of Standards,* and is intended to cover the more important papers of interest to the professional radio engineer which have recently appeared in periodicals, books, etc. The number at the left of each reference classifies the reference by subject, in accordance with the "Classification of Radio Subjects: An Extension of the Dewey Decimal System," Bureau of Standards Circular No. 385, obtainable from the Superintendent of Documents, Government Printing Office, Washington, D.C., for 10 cents a copy. The classification also appeared in full on pp. 1433–1456 of the August, 1930, issue of the PROCEEDINGS of the Institute of Radio Engineers.

The articles listed are not obtainable from the Government or the Institute of Radio Engineers, except when the publications thereof. The various periodicals can be secured from their publishers and can be consulted at large public libraries.

R000. RADIO (GENERAL)

R051 ×R130 E. L. Chaffee. Theory of Thermionic Vacuum Tubes (book). Published by McGraw-Hill Book Co., New York, N. Y. 1933. Price $6.00.

The present volume, which is the first of two, deals with the physics of the vacuum tube, and its use as a rectifier and amplifier.

R090 1922–1932—Dix ans de T.S.F. (1922–1932, ten years of radio.) *L'Onde Electrique*, vol. 11, pp. 397–570; November-December, (1932).

A historical survey of the progress in radio from 1922 to 1932.

R097 J. Zenneck. Valdemar Poulsen. *Hochfrequenz. und Elektroakustik*, vol. 41, pp. 113–115; April, (1933).

This is a brief article emphasizing the importance of and commemorating the invention of the Poulsen arc.

R100. RADIO PRINCIPLES

R113 F. Noether. Bemerkungen über das Ausbreitunggesetz für lange elektrische Wellen und die Wirkung der Heavisideschicht. (Observations on the propagation law for long electric waves and the effect of the Heaviside layer). *Elek. Nach. Tech.*, vol. 10, pp. 160–172; April, (1933).

Proceeding from the fact that surface waves cannot be produced from an antenna in a homogeneous atmosphere, it is asked whether such waves can be attained with the help of the Heaviside layer. The Watson type of waves is considered. A new meaning is said to be given to the Austin formula.

R113 The travel of wireless waves. *Electrician* (London), vol. 110, pp. 581–582; May 5, (1933).

A review of Sir Frank Smith's Kelvin Lecture, "How radio research has enlarged our knowledge of the upper atmosphere."

* This list compiled by Mr. A. H. Hodge and Miss E. M. Zandonini.

R113 F. J. W. Whipple. Relations between the combination coefficients of atmospheric ions. *Proc. Phys. Soc.* (London), vol. 45, pp. 367–380; May 1, (1933).

"The principal object of this paper is to put forward for consideration a formula $\eta_{12}-\eta_{10}=4\pi e\omega_1$ which indicates that the combination coefficient η_{10} for small ions and uncharged nuclei, and further that the difference between the two coefficients depends on the mobility ω_1 of the small ions. The experimental evidence for the formula is discussed, as well as possible applications."

R113.61 ×R113.5 R. C. Colwell. Cyclones, anticyclones, and the Kennelly-Heaviside layer. Proc. I.R.E., vol. 21, pp. 721–725; May, (1933).

Fading curves taken in Morgantown upon the signal of KDKA in Pittsburgh show an increase of intensity after nightfall provided a cyclonic area covers both cities or lies to the north of Morgantown. If a high pressure area covers both cities the night intensity does not increase above the day intensity and may even fall below it. These observations are explained by the theory that the Kennelly-Heaviside (E) layer is found at night in cyclonic regions but is not present in anticyclones. This theory is strongly supported by recent experiments of Ranzi on 100-meter waves.

R113.61 E. V. Appleton and R. Naismith. Weekly measurements of upper-atmospheric ionization. *Proc. Phys. Soc.* (London), vol. 45, pp. 389–398; May 1, (1933).

"A series of weekly measurements of the maximum ionization content of the Kennelly-Heaviside region of the ionosphere is reported and discussed. The ionization is found to be 2.2 times as intense on a summer noon as on a winter noon, and, in general slightly less in 1932 than in 1931. This reduction is considered to be probably due to the approach of sun spot minimum and, together with other evidence, suggests that the ionizing agency from the sun varies by as much as 60 per cent during the 11-year solar period." Thunderstorms as an ionizing agent are considered.

R113.61 J. A. Ratcliffe and E. L. C. White. An automatic recording method for wireless investigations of the ionosphere. *Proc. Phys. Soc.* (London), vol. 45, pp. 399–413; May 1, (1933).

An apparatus for continuous recording of the equivalent height of wireless echoes returned from the ionosphere is described. The Breit and Tuve method is employed, and both the transmitter and the time base at the receiver are synchronized with the alternating-current power supply. Some specimen records are reproduced, and are used to illustrate the normal diurnal variation of equivalent heights. Attention is directed to a common abnormal occurrence of increase of ionization in the lower or E region during the hours of darkness, without a corresponding increase in the upper or F layer. Reasons are for supposing that this may be due to the ionizing action of storm clouds, as suggested by C. T. R. Wilson.

R113.62 S. K. Mitra and H. Rakshit. Recording wireless echoes at the transmitting station. *Nature* (London), vol. 131, p. 657; May 6, (1933).

By using the same antenna for transmitter and receiver satisfactory reception of echo signals is accomplished at the transmitting station.

R113.7 N. Stoyko and R. Jouaust. Sur la vitesse apparente des ondes radio électriques courtes. (On the apparent velocity of short radio waves.) *Comptes Rendus*, vol. 196, pp. 1291–1292; May, (1933).

The apparent velocity of propagation of short waves is different from that of long waves. The difference is explained by the mechanism of propagation. Representative values of apparent velocity are: for short waves (15 to 35 meters) 269,700 km/sec. ± 4400; for long waves (frequency not given), 245,000 km/sec.

R120 C. E. Brigham. Antenna transmission line systems for radio reception. *Radio Engineering*, vol. 10, pp. 11–12, March; pp. 21–22, April, (1933).

Methods of using shielded lead-ins and coupling devices for receiver antennas are given.

R120 E. R. Sanders. The development of a transmitting antenna. *QST*, vol. 17, pp. 17–20; June, (1933).

Theory and methods of adjustments of transmitting antennas.

R120 F. Eppen and A. Goethe. Über die Schwundvermindernde Antenna des

Rundfunksenders Breslau. (On a broadcast antenna at Breslau which has a small amount of fading). *Elek. Nach. Tech.*, vol., 10, pp. 173–181; April, (1933).

Measurements on the antenna at Breslau have shown the correctness of the theory of the origin of fading. The Breslau antenna suppresses the space radiation to a 65-degree angle with the earth. The decreased space pattern increases the ground radiation, raising the horizontal field strength 26 per cent. The antenna is a quarter wave tower structure.

R130 E. C. S. Megaw. Electronic oscillations. *Jour. I.E.E.*, (London), vol. 72, pp. 313–325; April, (1933).

A summary is given of the existing knowledge of electronic oscillations and of their production in triode, diode, and magnetron circuits. The application of these oscillations to ultra short-wave communication is outlined, and apparatus used in a recent demonstration is described. A bibliography of 47 references is given.

R130 W. E. Benham. Electron conduction in thermionic valves. *Electrical Communication*, vol. 11, pp. 223–225; April, (1933).

The paper discusses the physical nature of the medium constituted by the cloud of electrons passing between cathode and anode of a thermionic vacuum tube.

R133 E. W. Helmholz. Experimentelle Prüfung der Theorie der Barkhausen-Schwingungen. (Experimental proof of the Barkhausen oscillation theory.) *Elek. Nach. Tech.*, vol. 10, pp. 181–193; April, (1933).

The Barkhausen theory is first briefly presented. An experimental apparatus for generating Barkhausen oscillations is then described and experimental results are given.

R140 G. Grammar. Circuits within circuits. *QST*, vol. 17, pp. 11–15; June, (1933).

Discussion of parasitic oscillations in neutralized amplifiers.

R148 W. R. Bennett. New results in the calculation of modulation products. *Bell Sys. Tech. Jour.*, vol. 12, pp. 228–243; April, (1933).

"A new method of computing products by means of multiple Fourier series is described. The method is used to obtain for the problem of modulation of a two-frequency wave by a rectifier a solution which is considerably simpler than any hitherto known."

R148.1 F. Massa. Permissible amplitude distortion of speech in an audio reproducing system. Proc. I.R.E., vol. 21, pp. 682–689; May, (1933).

This paper gives the experimental results of a brief study of permissible amplitude distortion in an audio reproducing system having a relatively flat frequency-response characteristic, from 80 to 14,000 cycles. The effect of distortion on the character of reproduction was observed when the transmission band was cut off at 5000 cycles, 8,000 cycles, and 14,000 cycles. It was found that the permissible amount of distortion is lower, the greater the frequency range of the reproducing system, and also that a certain amount of second harmonic is less objectionable than the same amount of third harmonic distortion.

R300. Radio Apparatus and Equipment

R320 Screened aerial down-leads. *Wireless World* (London), vol. 32, pp. 261–262; April 7, (1933).

Some useful and practical information is given on the subject of reducing interference by shielding lead-ins, and of overcoming the loss of sensitivity introduced through the use of a shielded lead-in.

R325.31 Wireless direction finding. *Electrician* (London), vol. 110, p. 513; April 21, (1933).

The Marconi-Adcock system of direction finding is briefly discussed. Installations are being made at Pulham, Norfolk, and Lympne, Sussex. This sytem is said to eliminate night errors. The receiver has sufficient amplification to be used with airplane transmitters 200 miles away.

R330 W. P. Koechel. The role of vacuum-tubes in a tube factory. *Electronics*, vol. 6, pp. 121–124; May, (1933).

The applications of vacuum tubes in the Ken-Rad manufacturing lant are described.

R330 E. C. S. Megaw. An investigation of the magnetron short-wave oscillator. *Jour. I.E.E.*, (London), vol. 72, pp. 326–348; April, (1933).

The possible methods of utilizing magnetrons to generate short-wave oscillations are indicated and the more important results of previous workers are described. The theoretical basis of "electronic" and "dynatron" oscillations is discussed. Results of an extended experimental investigation are given. The purpose of this research was to develop a tube that would deliver considerable power at very high frequency. An output of 60 milliwatts is obtained at 2.3-meter wavelength.

R330 C. B. DeSoto. The dial-coded universal tube checker and circuit analyzer. *QST*, vol. 17, pp. 21–23; June, (1933).

Details of set.

R331 H. W. Parker and F. J. Fox. The spray shield tube. Proc. I.R.E., vol. 21, pp. 710–720; May, (1933).

A metal coating is applied to a vacuum tube covering the tube envelope and the shell of the base by successive operations of sand blasting and spraying metal from an oxy-acetylene blow torch which produces a coating which is in intimate contact with the glass envelope. A negative potential is applied to the metal spray shield when the tube is operated which prevents cathode rays, which are not intercepted by the anode, from reaching the interior surface of the glass envelope. Secondary emission of electrons from the glass wall is avoided. Localized heating of the glass is prevented and the fluctuation component of anode current is suppressed.

R355.8 A. A. Collins. Getting quality performance with class B modulation. *QST*, vol. 17, pp. 12–17; May, (1933).

Practical design and operating data for best tube combinations for quality performance.

R356.3 J. B. Epperson. Three-phase transformer connections and their application to high voltage rectifying circuits. *Radio Engineering*, vol. 13, pp. 14–16; May, (1933).

The usual types of three-phase transformer connections as used in high voltage rectifiers are given.

R360 H. F. Olson and F. Massa. A high quality ribbon receiver. Proc. I.R.E.,
×R160 vol. 21, pp. 673–681; May, (1933).

The ribbon receiver consists of a ribbon diaphragm in a magnetic field. In order that the ratio of pressure in the ear cavity to the applied voltage shall be independent of the frequency, the ratio of the amplitude of the ribbon to the applied voltage must be independent of the frequency. This is accomplished by employing an acoustic system consisting of two resonant circuits. The amplitude response of this receiver has a maximum variation of $\pm 2\frac{1}{2}$ decibels in the range 30 to 10,000 cycles.

R361 J. H. Miller. The use of instruments in radio receiver manufacturing. *Radio Engineering*, vol. 13, pp. 20–21; May, (1933).

The measurements, tests and the instruments used in making radio receiving sets are discussed.

R361 C. F. Hadlock. Improving the 56-mc receiver. *QST*, vol. 17, pp. 23–26; May, (1933).

Constructional details of two new sets for the ultra high frequencies.

R361 H. B. Fischer. A crystal-control superheterodyne receiver. *Bell Lab. Record*, vol. 11, pp. 273–278; May, (1933).

A receiver, Western Electric 12-A, which is designed for airplane use is described.

R363 L. Tulaukas. Bridge-type push-pull amplifiers. *Electronics*, vol. 6, pp. 134–135; May, (1933).

Several modifications of the push-pull circuit are given. The chief advantages are that the modified circuits demand less space and cost.

R363.2 L. C. Verman. A high gain audio-frequency amplifier. *Rev. Sci. Inst.*, vol. 4, pp. 153–156; March, (1933).

A three-stage high gain amplifier with an amplification of 200 per stage is described. The gain-frequency characteristics are given for power as well as voltage amplification.

R365.21 I. J. Cohen. Automatic gain control of radio receivers. *P. O. Elec. Eng. Jour.* (London), vol. 26, pp. 58–59; April, (1933).

The need and operation of automatic gain control of high-frequency receivers are discussed.

R365.21 C. B. Fischer. Automatic volume control for radio receivers. *Wireless Engineer and Experimental Wireless* (London), vol. 10, pp. 248–254; May, (1933).

"The value of automatic volume control for radio receivers is discussed, with a description of American development up to the present time. Details of a typical circuit are given and the curves of its operation are shown. Various limitations on the circuit arrangement are considered."

R382 W. J. Polydoroff. Ferro-inductors and permeability tuning. Proc. I.R.E., vol. 21, pp. 690–709; May, (1933).

Brief analysis indicates that tuning by variation of inductance in such a manner that L/R of the circuit is kept constant results in constant selectance and amplification throughout the tuning range. Very finely divided and compressed magnetic-core material has been developed for this purpose. Between the prescribed limits of frequency range, iron cores can be so designed as to produce simultaneous inductance and resistance variations of the order of an air-core inductance. Constructional details of variable ferro-inductors are given, together with their behavior and application in radio circuits.

R384 A wavemeter with alternative close and open scales. *Wireless Engineer and Experimental Wireless* (London), vol. 10, pp. 255–257; May, (1933).

"An absorption wavemeter is described which can be used (a) with a wide frequency range; (b) with a restricted range and correspondingly greater openness of scale. Observations are made on the calibration of such wavemeters, and their use in connection with receiving apparatus. The use of the principle in receiver tuning is noted."

R384 W. H. F. Griffiths. The simplification of accurate measurement of radio frequency. *Wireless Engineer and Experimental Wireless* (London), vol. 10, pp. 239–247; May, (1933).

The successive stages of improvement which have been effected in resonant circuit wavemeters are briefly described. An extraordinarily accurate and stable design of an oscillating wavemeter working on the dynatron principle is described. Curves are given showing the variation of frequency with supply voltage. An accuracy of 1 part in 20,000 is claimed together with a stability of 1 part in 10^5. Range 150 to 10,000 meters.

R385.5 ×534 H. F. Olson. On the collection of sound in reverberant rooms with special reference to the application of the ribbon microphone. Proc. I.R.E., vol. 21, pp. 655–672; May, (1933).

The effective reverberation of collected sound may be expressed as the ratio of direct to generally reflected sound. Direction sound collectors discriminate against reverberation and other undesirable sounds by increasing the ratio of direct to generally reflected sound. The ribbon microphone, consisting of a light metallic ribbon suspended in a magnetic field and freely accessible to air vibrations from both sides, exhibits direction characteristics which are independent of the frequency.

R388 A. B. DuMont. Recent developments in cathode-ray tubes and associated apparatus. *Radio Engineering*, vol. 10, pp. 15–19, March; pp. 20–21, April, (1933).

The general development of the cathode-ray tube, its characteristics and uses are discussed.

R400. Radio Communication Systems

R420 H. H. Beverage; H. O. Peterson; C. W. Hansell. Trans-oceanic radio communication. *Electrical Engineering*, vol. 52, pp. 331–337; May, (1933).

The problems, demands, methods and apparatus used in transoceanic radio communication are discussed.

R500. Applications of Radio

R520 D. K. Martin. New radio telephone equipment for transport planes. *Bell Lab. Record*, vol. 11, pp. 262–266; May, (1933).

A brief general description of a communication system for transport planes. One radio transmitter and two receivers constitute the apparatus.

R522.1 W. C. Tinus. A three-frequency radio telephone transmitter. *Bell Lab. Record*, vol. 11, pp. 267–272; May, (1933).

A description of the radio transmitter 13-A for airplanes.

R526.2 C. F. Green and H. I. Becker. Radio aids to air navigation. *Electrical Engineering*, vol. 52, pp. 307–313; May, (1933).

A radio compass is described. This compass uses a loop and vertical antenna. The radio-frequency energy from the loop and vertical wire are converted to direct current which will reverse polarity when the loop is turned from bucking to a boosting position. This is accomplished by a modulating audio frequency. A nonlinear resistor gives a visual indication of plane relative to course. A method of using this in connection with a magnetic compass is given.

R526.3 S. P. Johnson. Down the landing beam. *Aviation*, vol. 82, pp. 162–163; May, (1933).

A popular description of the fog landing system of the Department of Commerce which was recently demonstrated at Newark, N. J.

R800. Nonradio Subjects

621.313.7 ×535.38 O. L. Grondahl. The copper-cuprous oxide rectifier and photoelectric cell. *Rev. Modern Physics*, vol. 3, pp. 141–168; April, (1933).

This is an extensive and careful review of the known facts relative to the copper-cuprous oxide rectifier and photoelectric cell. Many practical data are given. Methods of manufacture and characteristics are discussed. The various theories that have been proposed are reviewed. An excellent bibliography of 141 references is given.

621.385 The reproduction of orchestral music in audition perspective. *Bell Lab. Record*, vol. 11, pp. 254–261; May, (1933). *Electronics*, vol. 6, pp. 118–120; May, (1933).

The requirements of perfect audio-frequency reproduction are discussed in connection with a description of the reproduction in Washington, D.C., of a program produced in Philadelphia, Pa. Land lines were used. The full frequency range of nine octaves was reproduced.

CONTRIBUTORS TO THIS ISSUE

Andrew, Victor J.: Born August 31, 1902, in Medina County, Ohio. Received B.S. degree, College of Wooster, 1926; Ph.D. degree, University of Chicago, 1932. Radio Engineer, Signal Corps Radio Laboratories, 1926–1927; Westinghouse Electric and Manufacturing Company, 1928–1930; X-ray engineer, Westinghouse Lamp Company, 1933. Associate member, Institute of Radio Engineers, 1925; Member, 1930,

Brainerd, John Grist: See PROCEEDINGS for January, 1933.

Callendar, M. V.: Born October 5, 1906, at London, England. Received B.A. degree in Physics, Cambridge University, 1928. Research physicist, Lissen, Limited, 1928; in charge, radio development department, 1932 to date. Associate member, Institute of Radio Engineers, 1933.

Caporale, Peter: Born October 8, 1906, at Philadelphia, Pennsylvania. Received B.S. degree in electrical engineering, 1928; M.S. degree in electrical engineering, 1929, Moore School of Electrical Engineering, University of Pennsylvania. Instructor in mathematics, Temple University, 1928–1929; instructor in electrical engineering, Drexel Institute, 1930; instructor in basic and high-frequency electric current theory in out-of-hour courses, RCA Victor Co. Student engineer, Radio Corporation of America, 1929; work in electro-acoustics, RCA Victor Co., 1930–1932; consulting engineer, Electro-Acoustical Engineering Company of America, 1932 to date. Associate member, A. I. E. E.; Acoustical Society of America; Member, American Association for the Advancement of Science; Associazione Elettrotecnica Italiana. Associate member, Institute of Radio Engineers, 1928.

De Cola, Rinaldo: Born September, 1907, at Niles, Ohio. Amateur radio operator, 1922 to date. Commercial radio operator on the Great Lakes, summers, 1924–1929; engaged in private research, study, and technical writing, 1924–1929; chief radio engineer, Apex Electrical Manufacturing Company; consultant to The Victoreen Radio Company and the Workrite Radio Corporation, 1929–1931; research engineer, Tung-Sol Radio Tubes, Inc., 1931–1932; consulting engineer, 1932 to date. Nonmember, Institute of Radio Engineers.

Dellinger, J. H.: Born July 3, 1886, at Cleveland, Ohio. Educated at Western Reserve, 1903–1907; received A.B. degree, George Washington University, 1908; Ph.D. degree, Princeton University, 1913; D.Sc. degree, George Washington University, 1932. Instructor, physics department, Western Reserve, 1906–1907. Joined staff, Bureau of Standards as physicist, 1905; Chief of Bureau's Radio Section, 1919 to date; Chief of Radio Section, Research Division, Aeronautics Branch, Department of Commerce, 1926 to date. U. S. representative at numerous international radio and electrical conference, 1921 to date. Vice President, Institute of Radio Engineers, 1924; President, 1925. Fellow, Institute of Radio Engineers, 1923.

Eckersley, T. L.: Born December 27, 1886. Received B.S. degree in electrical engineering, University College, London, 1908. National Physical Laboratory, 1909; Cambridge, 1910; research at Cavendish; appointed inspector in Egyptian Government Survey Department, 1912. Served during war with R. E. Wireless Intelligence work; research department, Marconi Company, 1919 to date. Nonmember, Institute of Radio Engineers.

Glessner, J. M.: Born April 21, 1905, at Berkeley, California. Received B.S. degree, University of California, 1927. Lighting section, research laboratories, General Motors Corporation, 1927–1929; research division, Crosley Radio Corporation, 1930 to date. Associate member, Institute of Radio Engineers, 1930.

Groszkowski, Janusz: Born March 21, 1898, at Warsaw, Poland. Received Doctor of Technical Science degree, Warsaw Polytechnical High School, 1927; professor, Warsaw Polytechnical High School, 1929 to date. Director, Radio Institute, Poland, 1928 to date. Member, Institute of Radio Engineers, 1931.

Grover, F. W.: Born September 3, 1876, at Lynn, Massachusetts. Received B.S. degree, Massachusetts Institute of Technology, 1899; M.S. degree, Wesleyan University, 1901; Ph.D. degree, George Washington University, 1907; Ph.D. degree, University of Munich, 1908. Instructor, Wesleyan University, 1899–1901; Lafayette College, 1901–1902; laboratory assistant, Bureau of Standards, 1902–1904; assistant physicist, 1904–1907, 1908–1911; professor of physics, Colby College, 1911–1920; assistant professor, electrical engineering, Union College, 1920–1922; associate professor, 1922 to date. Engaged in educational work for U. S. Signal Corps, summers, 1917–1918. Consultant, Bureau of Standards, 1918 to date. Member, Institute of Radio Engineers, 1917.

Hutcheson, J. A.: Born January 21, 1905, at Park River, North Dakota. Received B.S. degree, University of North Dakota, 1926. Radio engineering department, Westinghouse Electric and Manufacturing Company, 1926 to date. Member, Sigma Xi. Associate member, Institute of Radio Engineers, 1928; Member, 1930.

Kilgour, C. E.: Born March 29, 1886, at Detroit, Michigan. Received C.E. degree, University of Cincinnati, 1910. Railroad construction, Atcheson, Topeka, and Santa Fe, 1910; industrial engineer, E. M. Chace, 1916–1917; mechanical and radio design, Crosley Manufacturing Company, 1917–1923; chief engineer in charge of research, Crosley Radio Corporation, 1923 to date. Associate member, Institute of Radio Engineers, 1925; Member, 1930.

Le Corbeiller, Philippe: Born January 11, 1891, at Paris, France. École Polytechnique, 1911–1913; École Supérieure d'Electricité, 1919. Received Ph.D. in mathematics, Paris, 1926. Engineer, Ministry of Post, Telegraph, and Telephone, 1920; National Radio Laboratory, 1933. Nonmember, Institute of Radio Engineers.

Namba, Shogo: See Proceedings for February, 1933.

Tsukada, Taro: Born May 18, 1907, at Tokyo, Japan. Studied electrical engineering, Tokyo Higher Technical School, 1928–1931. Radio section, Electrotechnical Laboratory, Ministry of Communications, (Japan), 1931 to date. Nonmember Institute of Radio Engineers.

Van der Pol, Balthasar: Born January 27, 1889, at Utrecht, Holland. Studied experimental physics with Professor J. A. Fleming, London, 1916–1917; with Professor Sir. J. J. Thomson, Cambridge, 1917–1919. Conservator Physical Laboratory, Teyler's Stichting, Haarlem, Holland, as assistant to Professor Dr. H. A. Lorentz, 1919–1922. Received Doctor of Physics degree, Utrecht, 1920. Member, scientific staff, N. V. Philips' Glowlamps, Eindhoven, Holland, 1922–1925; chief, scientific research, N. V. Philips' Radio, 1925 to date. One of the founders and for many years president of "Het Nederlandsch Genootschap;" charter member, Technical Committee, Union Internationale de Radiodiffusion; president, Commission de Radiophysique, Union Radio Scientifique Internationale; represented as representative, U.I.R. European radio broadcasting, Washington International Radio Conference, 1927, represented as representative, U.R.S.I. and U.I.R., Conference of the Comité Consultatif International Radio, The Hague, 1929, Copenhagen, 1932; Madrid, 1932. Member, Institute of Radio Engineers, 1920; Fellow, 1929.

Williams, N. H.: Received Ph.D. degree, University of Michigan, 1912. Research Laboratory, General Electric Company, 1923–1924; physics department, University of Michigan, 1908 to date; professor of physics, 1919 to date. Associate member, Institute of Radio Engineers, 1930.